Chapter Eight Further Applications of Integration

Section 8.1 Arc Length

Given a function f(x) is continuous over interval [a,b]. How to find arc – length of f(x) over [a,b]

<u>Def</u>: If f' is continuous on [a, b], then the length of the curve $y = f(x)$ for $a \le x \le b$, is $L = \int_a^b \sqrt{1 + f'(x)}dx$

Or $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ If the function is in term of x Or $L = \int_a^b \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$ If the function is in term of y.

General form:

Ex: Find the arc – length of the following:

a) $y = x^{2/3}$ from (1, 1) to (8, 4)

b) $y = \frac{x^2}{4} - \frac{\ln x}{2}$ from x = 1 to x = 2.

c) $y = \ln\left(\frac{e^x + 1}{e^x - 1}\right)$ *from* $x = 1$ *to* $x = 2$

d) $y = \frac{x^3}{3} + x^2 + x + \frac{1}{4x + 4}$ *for* $0 \le x \le 2$

e) $x = \frac{y^{3/2}}{3} - y^{1/2}$ from y = 2 to y = 9.

f) $x = \int_0^y \sqrt{\sec^4 t - 1}\,dt \ \ for\ -\frac{\pi}{4} \le y \le \frac{\pi}{4}$

Section 8.2 Area of a Surface of Revolution

Ex: Find the area of the surface formed by rotating about the x axis the arc $y = \frac{x^3}{3}$ from x = 0 to x =2.

Ex: Find the surface area of the arc of the curve $x = \frac{y^3}{6} + \frac{1}{2y}$ from (2/3, 1) to (14/3, 3),

a) Rotates about the y – axis.

b) Rotates about the x – axis.

Ex: The curve $y = \sqrt{2x - x^2}$ for $0.5 \le x \le 1.5$ is rotated about the x – axis. Find its surface area.

Ex: Show that the surface area of a sphere of radius r is $4\pi r^2$

Ex: The region bounded by $y = \frac{1}{x}$; $y = 0$ *and* $x \geq 1$ is rotated about the x – axis.

a) Find its volume.

b) Find it surface area.