Chapter 10 Parametric Equations and Polar Coordinates

Section 10.1 *Curves Defined by Parametric Equations*

Def: Let $x = f(t)$ and $y = g(t)$, where f and g are two functions whose common domain is some vertical I. the collection of points defined by $(x, y) = (f(t), g(t))$ is called a plane curve. The equations $x = f(t)$ and $y = g(t)$ where t is in I, are called parametric equations of the curve. The variable t is called a parameter.

Ex: Sketch the graph of the following parametric equations and indication the direction on the graph.

a) $\begin{cases} x = 3t^2 \\ y = 2t \end{cases}$; $for \quad -2 \le t \le 2$

b) $\begin{cases} x = 2\cos t + 2 \\ y = 3\sin t - 1 \end{cases}$

c) $\begin{cases} x = 3\sin^2 t - 1 \\ y = 2\cos t + 3 \end{cases}; -2 \le t \le 2$

d) $\begin{cases} x = (v_0 \cos\theta)t \\ y = -\frac{1}{2}gt^2 + (v_0 \sin\theta)g + h \end{cases}$ (Time as a Parameter: Projectile Motion)

Ex: Suppose that Jim hit a golf ball with an initial velocity of 150 ft/sec. at an angle of 30 degree to the horizontal.

a) Find parametric equations that describe the position of the ball as a function of time.

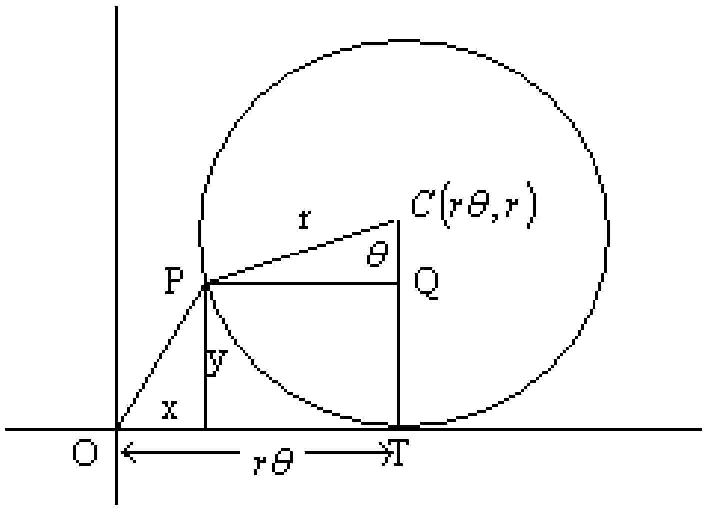
b) How long is the golf ball in the air?

c) When is the ball at its maximum height? Determine the maximum height of the ball.

d) Determine the distance that the ball traveled.

Ex: The curve traced out by a point P on the circumference of a circle as the circle rolls along a straight line is called a cycloid. If the circle has radius r and rolls along the x-axis and if one position of P is the origin, find the parametric equations for the cycloid.

Sol:



<u>Section 11.2</u> ***Calculus with Parametric Curves:***

Given a parametric equation: $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$ *for* $t \in I$

1. First derivative:

2. Second derivative:

Ex: Find point(s) where tangent lines to $\begin{cases} x = t^2 + t - 6 \\ y = t^2 - 3t - 4 \end{cases}$ is either vertical or horizontal

Ex: A curve C is defined by the parametric equations $x = t^2; y = t^3 - 3t$

a) Show that C has two tangents as the point (3,0) and find their equations.

b) Find the points on C where the tangent is horizontal or vertical.

c) Determine where the curve is concave upward or downward.

d) Sketch the curve with direction.

Ex: a) Find the tangent to the cycloid $x = r(\theta - \sin\theta);\ y = r(1 - \cos\theta)$ at the point $\theta = \pi/3$

b) At what points is the tangent horizontal? When is it vertical?

Areas:

Ex: Find the area under one arch of the cycloid $x = r(\theta - \sin\theta)$, $y = r(1 - \cos\theta)$

Arc Length:

Ex: Find the length of one arch of the cycloid $\begin{cases} x = r(t - \sin t) \\ y = r(1 - \cos t) \end{cases}$, Ex: Find the length of the following curve $\begin{cases} x = \cos t + \ln\left(\tan \frac{1}{2} t\right) \\ y = \sin t \end{cases}$; for $\pi / 4 \leq t \leq 3\pi / 4$

Surface Area:

Ex: Find the surface area of the following which rotated about the indicated axis.

a) $\begin{cases} x = e^t - t \\ y = 4e^{t/2} \end{cases}$; $0 \le t \le 1$ rotated about the x – axis.

b) $\begin{cases} x = \ln(\sec t + \tan t) - \sin t \\ y = \cos t \end{cases}$; $0 \le t \le \frac{\pi}{3}$; *about the* $x-$*axis*