Section 10.2   Polar Coordinates

Def: A point P is represented by the order pair (r,θ) where r is the distance from the point to the origin, and theta is the angle from the x-axis to the line connecting the point and the origin.

So for any point $(x,y) \Rightarrow (r,\theta) \Rightarrow \begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

Identities: $x^2 + y^2 = r^2 ; \tan\theta = \dfrac{y}{x}$

Ex: Represent the following point in Cartesian coordinate to the polar coordinates.

      a) $\left(1,\sqrt{3}\right)$ b) $\left(1,-1\right)$

Ex: Locate the following points in polar coordinates.
           a) $(5, \pi/4)$ b) $(-2, 7\pi/6)$

Ex: Convert the following into rectangular coordinates:
        a) $r^2 = 3r \sin\theta - 4\cos\theta$

        b) $r^3 = 2r \cos\theta - 5r \sin\theta$

Ex: Convert to polar coordinate:
   a) $x^2 + y^2 = 25$ b) $7x - 5y^2 = 4$

Polar Curves:

The graph of a polar equation $r = f(\theta)$ or more generally, $F(r,\theta) = 0$ consists of all points P that have at least one polar representation (r,θ) whose coordinates satisfy the equation.

Ex: Sketch the graph of the following:
   a) $r = \theta$ b) $r = 1/\theta$

   c) $r = 3$ d) $\theta = \pi / 3$

e) $r = 2\cos\theta$

f) $r = 5\sin(\theta)$

l) $r = -5\cos(2\theta)$

m) $r = 2\cos(3\theta)$

n) $\quad r = 3 - 2\cos\theta$

o) $\quad r^2 = -9\cos(2\theta)$

p) $\quad r^2 = -4\sin(3\theta)$

How to sketch $r = a \pm b \cos\theta$ *and* $r = a \pm b \sin\theta$

Tangents to Polar Curves

To find a tangent line to a polar curve $r = f(\theta)$, we regard θ as a parameter and write its parametric equations as

$$x = r\cos\theta = f(\theta)\cos\theta \text{ and } y = r\sin\theta = f(\theta)\sin\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} = \frac{\dfrac{dr}{d\theta}\sin\theta + r\cos\theta}{\dfrac{dr}{d\theta}\cos\theta - r\sin\theta}$$

Ex: a) For the cardioid $r = 1 + \sin\theta$, find the slope of the tangent line where $\theta = \pi/3$

b) Find the points on the cardioid $r = 4 - 4\cos\theta$ where the tangent line is horizontal or vertical.