Section 10.4 Area and Lengths in Polar Coordinates

<u>Ex:</u> Find the area enclosed by one loop of the four-leave rose $r = \cos(2\theta)$.

Ex: Find the area of the inner loop of $r = 4 - 8\cos\theta$

<u>Ex</u>: Given $r_1 = 4\sin(\theta)$ *and* $r_2 = 2$. Set up integral(s) for area

a) Inside r1 / outside r2

b) Inside r2 / outside r1

c) Inside both r1 and r2.

Ex: Given $r_1 = -5\sin\theta$ *and* $r_2 = 5 + 5\sin\theta$. Sketch and set up integral(s) for area

a) Inside r1 / outside r2.
b) Inside r2 / outside r1
c) Inside both r1 and r2.

Ex: Find the area of the region that lies inside the circle $r = 3\sin\theta$, and the cardioid $r = 1 + \sin\theta$.

Ex: Sketch and find the area of the region that lies inside the circle $r = 1$ and outside the cardiod $r = 1 - \cos\theta$

Arc – Length for polar coordinate:

Ex: Find the arc – length of the spiral $r = e^{\theta} \; for \; 0 \le \theta \le \pi$

Ex: Find the arc – length of the cardioid $r = 1 - \cos\theta \;\; for \;\; 0 \le \theta \le 2\pi$

Area of a surface of revolution: Given $r = f(\theta)$ has a continuous first derivative for $\alpha \le \theta \le \beta$, and if the point $P:(r,\theta)$ traces trace the curve $r = f(\theta)$ exactly once for $\alpha \le \theta \le \beta$, then area of the surfaces generated revolving the curve about the x – and y – axes are given

1. Rotated about the x – axis. $S = \int_{\alpha}^{\beta} 2\pi r \sin\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \quad for\ y \ge 0$

2. Rotated about the y – axis $S = \int_{\alpha}^{\beta} 2\pi r \cos\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta \quad for\ x \ge 0$

<u>Ex:</u> Find the area of the surface generated by revolving the right – hand loop of the lemniscate $r^2 = \cos(2\theta)$ about the y – axis.