The Comparison Tests

The Comparison Test: Suppose $\sum_{n=1}^{\infty} a_b$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms.

a) If $\sum_{n=1}^{\infty} b_n$ is convergent and $a_n \le b_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ is also convergent.

b) If $\sum_{n=1}^{\infty} b_n$ is divergent and $a_n \ge b_n$ for all n, then $\sum_{n=1}^{\infty} a_n$ is also divergent.

a) $\displaystyle\sum_{n=1}^{\infty} \frac{7n^2 + 5n^3 + 4}{\sqrt{4n^{12} + 5n + 4}}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{4n^3 + 5n + 8}{\sqrt{n^8 + 5n + 12}}$

c) $$\sum_{n=1}^{\infty} \frac{n^2 + \sin(7n + \pi)}{n^5 + 3n + 5}$$

d) $$\sum_{n=1}^{\infty} \frac{n^2 + 4^n + 4}{n^5 4^{n+1}}$$

e) $$\sum_{n=1}^{\infty} \frac{2n^2 + 3\cos^2(n) + 4}{\sqrt{n^5 + 5n + 7}}$$

The Limit Comparison Test:

Suppose that $\sum_{n=1}^{\infty} a_b$ and $\sum_{n=1}^{\infty} b_n$ are series with positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c is a ***finite number*** and $c > 0$, then either both series converge or both diverge.

Suppose $a_n > 0$ *and* $b_n > 0$ for all $n \geq N$

1. $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0 \Rightarrow$ both converge or both diverge.

2. $\lim_{n\to\infty} \frac{a_n}{b_n} = 0 \ \textit{and} \ \sum b_n \ \textit{converges, then} \ \sum a_n \ \textit{converges}$

3. $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty \ \textit{and} \ \sum b_n \ \textit{diverges, then} \ \sum a_n \ \textit{diverges}$

Ex: Test for convergence

a) $\sum_{n=1}^{\infty} \frac{7n^3 - n + 4}{\sqrt{49n^{10} - 9n + 4}}$

b) $$\sum_{n=1}^{\infty} \frac{7n - n + 4}{\sqrt{2n^4 + 5n - 2}}$$

c) $$\sum_{n=1}^{\infty} \frac{1}{2^n - 1};$$

d) $$\sum_{n=1}^{\infty} \frac{n+1}{n2^n}$$