Section 11.8 ***Power Series***

Def: A power series is a series of the form $\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots$

General form: $\sum_{n=0}^{\infty} c_n (x-a)^n$: power series centered at a or a power series about a.

We know that $\sum_{n=0}^{\infty} x^n$ is convergence if and only if $|x| < 1$

Ex: Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{n}{2^n}(x-1)^n$

Ex: Find interval of convergence:

a) $\sum_{n=0}^{\infty} n!x^n$

b) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

Theorem: For a given power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ there are only three possibilities:

i) The series converges only when x = a.
ii) The series converges for all x.
iii) There is a positive number R such that the series converges if $|x-a| < R$ and diverges if $|x-a| > R$

Ex: Determine interval of convergence:

a) $\sum_{n=1}^{\infty} \frac{n}{2^n}(x-1)^n$

b) $\sum_{n=0}^{\infty} 2^n \cos^n x$

c) $$J_0 = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n} (n!)^2}$$

d) $$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$$

e) $$\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$$

Section 12.9 Representations of Functions as Power Series

$$\sum_{n=1}^{\infty} ax^{n-1} = \frac{a}{1-x} \text{ for } |x| < 1$$

$$\sum_{n=0}^{\infty} ax^{n} = \frac{a}{1-x} \Rightarrow \sum_{n=0}^{\infty} x^{n} = \frac{1}{1-x} \text{ for } |x| < 1$$

<u>Ex</u>: Express the following expression as the sum of a power series and find the interval of convergence.

a) $f(x) = \dfrac{1}{1+2x^2}$

b) $f(x)=\dfrac{9x-7}{x^2-x-6}$ about the center $x=8$

Theorem: If the power series $\sum_{n=0}^{\infty} c_n (x-a)^n$ has radius of convergence $R>0$, then the function f defined by

$f(x)=c_0+c_1(x-a)+c_2(x-a)^2+c_3(x-a)^3+\ldots=\sum_{n=0}^{\infty} c_n (x-a)^n$ is differentiable (and therefore continuous) on the interval $(a-R,a+R)$ and

i) $$f'(x)=c_1+2c_2(x-a)+3c_3(x-a)^2+\ldots=\sum_{n=1}^{\infty} nc_n (x-a)^{n-1}$$

ii) $$\int f(x)dx=C+c_0(x-a)+c_1\frac{(x-a)^2}{2}+c_2\frac{(x-a)^3}{3}+\ldots=C+\sum_{n=0}^{\infty} c_n\frac{(x-a)^{n+1}}{n+1}$$

Ex: Find a power series of representation and interval of convergence

a) $f(x) = \ln(1 + 4x^7)$

b) $f(x) = \dfrac{1}{(1 - 5x^3)^2}$

c) $f(x) = \tan^{-1} x$

Ex: Evaluate the following as a power series:

a) $\int \frac{x^{1.4}}{2+x^7}\,dx$