Section 6.1 ***Area between curves***

Given two functions $f(x)$ *and* $g(x)$ *over* $[a,b]$. How to find the area between the two curves:

Ex: Find the area bounded between

a) $y = x^2 - x - 6$ *and* $y = 2x - 2$ *for for* $-2 \le x \le 4$

b) $x = -2y^2 - 3y - 2$ *and* $x = y - 8$ *for* $-3 \le y \le 2$

c) $y = x^2 - x - 2$ *and* $y = -x^2 - x + 6$ *for* $-3 \le x \le 2$

Ex: Find k so that $x = k$ bisects the area bounded by $y = \frac{1}{x^2}$ *and* $x - axis$ $1 \le x \le 4$

Section 6.2 *Volumes (Dish method, Cross Section)*

From volume for basic shapes such as box, cylinder, sphere… We can find volume of irregular shape but uniform height, and then we find volume by the area of the base times the height.

Def: Let S be a solid that lies between x = a and x = b. If the cross-sectional area of S in the plane P, through x and perpendicular to the x-axis, is A(x), where A is a continuous function, then the volume of S is

$$V = \lim_{n\to\infty} \sum A\left(x_i^*\right)\Delta x = \int_a^b A(x)dx$$

Ex: Find the volume of the following region bounded by

a) $y = \sqrt{x}$, $y = 0$ *and* $x = 4$, rotated about the x – axis.

b) $y = x^3$ *and* $y - axis$ *for* $0 \le y \le 1$

c) $y = \sqrt{x}$ *and* $y = x$ *about* $x = 2$

d) $y = x^2 - x - 6$ *and* $y = 2x + 4$ *about* $y = 14$

Ex: Show that the volume of a sphere of radius r is $V = \frac{4}{3}\pi r^3$

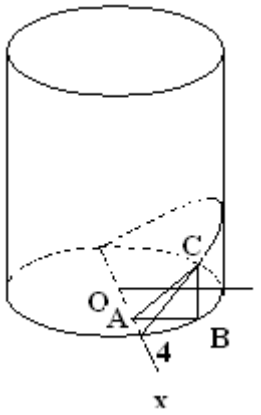
Parallel Cross Sections

Ex: Find the volume of a pyramid whose base is a square with side L and whose height is H.

Ex: The base of a solid is bounded by $\left\{(x,y)\middle|4x^2+9y^2\le 36\right\}$. All parallel cross sections are squares which perpendicular to the base and the x – axis. Find the volume of the solid.

Ex: The base of a solid is bounded by $y=x^2$ *and* $y=x$. All parallel cross sections are semi-circle perpendicular to the base and the x-axis.

Ex: A wedge is cut out of a circular cylinder of radius 4 by two planes. One plane is perpendicular to the axis of the cylinder. The other intersects the first at an angle of 30 degree along the diameter of the cylinder. Find the volume of the wedge.



6.3 ***Volume by cylindrical shell method.***

Ex The region bounded by the following curves, sketch and set up integrals for volume:

a) $V = 6 + x - x^2$ *and* $y = 2x - 6$

i) rotates about the line $x = 3$

ii) rotates about the line $y = -14$

b) The region bounded by $x = -y^2 + 2y + 3$ *and* $x + y = -1$

i) rotated about the line $y = 4$

ii) rotated about the line $x = -5$