Section 7.8 Improper Integrals

<u>Ex:</u> Evaluate the integral: $\int_0^2 \frac{1}{(x-1)^2}\,dx =$

Let's check the graph:

Type 1: Improper Integrals (Infinite Limits of Integration)

1. If f is continuous on the interval $[a,\infty]$

2. If f is continuous on the interval $(-\infty,b]$

3. If f is continuous on the interval $(-\infty,\infty)$

Ex: Evaluating the following improper integrals:

a) $\displaystyle\int_0^{\infty} \frac{x}{\left(4x^2+1\right)^{5/2}}\,dx$

b) $\displaystyle\int_1^{\infty} \frac{x^2}{\sqrt[3]{7x^3+1}}\,dx$

c) $\int_{-\infty}^{\infty} \frac{x}{x^4+9} dx$

d) $\int_{-\infty}^{0} \frac{x}{e^{1-3x^2}} dx$

P-Test Theorem: For what values of p is the integral $\int_1^\infty \frac{1}{x^p}\,dx$ convergent?

Ex: Test for convergence / divergence:

a) $\int_1^\infty \frac{\sqrt{x^5}}{x^3}\,dx$

b) $\int_1^\infty \frac{x}{\sqrt[3]{x^5}}\,dx$

Type 2*:** ***Discontinuous Integrands

1. If f is continuous on the interval [a, b) and approaches infinity at b.

2. If f is continuous on the interval (a, b] and approaches infinity at a.

3. If f is continuous on the interval [a,b], except for some c in (a, b).

Ex: Evaluate the following improper integrals

a) $\int_0^2 \frac{x^2}{\sqrt[3]{x^3-8}}dx$

b) $\int_{2/3}^1 \frac{1}{\sqrt[3]{3x-2}}$

c) $\int_0^{\frac{\pi}{6}} \tan(3x)\,dx$

d) $\int_3^6 \frac{x}{x^2-25}\,dx$

<u>Comparison test for improper integrals</u>:

Suppose that f and g are continuous functions with $f(x) \geq g(x) \geq 0$ for $a \geq 0$

a) If $\int_a^\infty f(x)dx$ is convergent, then $\int_a^\infty g(x)dx$ is convergent.

b) If $\int_a^\infty g(x)dx$ is divergent, then $\int_a^\infty f(x)dx$ is divergent.

Ex: Test for convergence / divergence:

a) $\int_0^\infty e^{-x^2}\,dx$

b) $\int_1^\infty \frac{7x^2+5x+2}{4x^7+3x^2+1}\,dx$

c) $\int_2^\infty \frac{4x^3+3x+2}{\sqrt{7x^9+x^8+3}}\,dx$

d) $\int_1^{\infty} \frac{1+e^{-x}}{x}\,dx$

e) $\int_1^{\infty} \frac{\sin(5x-3)+4\tan^{-1}(3x)}{x^3+3x+4}\,dx$

f) $\int_1^{\infty} e^{x^2+x+1} dx$

g) $\int_0^{\pi/2} \frac{1}{x\sin(x)} dx$

Ex: Find the volume of the following:

a) The region bounded by $y = e^{-x}$; $y = 0$; $for\ x \geq 0$ is rotated about the x – axis.

b) The region bounded by $y = \tan(x)$; $y = 0\ \ for\ \ 0 \leq x \leq \frac{\pi}{2}$ is rotated about the x – axis.