

Why?

Rules for implicit differentiation:

Variable:  $\frac{d}{dx}(x^n) = nx^{n-1}$

Function of x:  $y = f(x): \frac{d}{dx}(y^n) = ny^{n-1} \frac{dy}{dx} = ny^{n-1}y'$

Ex: Determine:  $\frac{dy}{dx}$  of the following:

a)  $x^2 + y^2 = 25$

b)  $\sin^3(x^2 + y^3) - e^{3x+y} = 4$

***Product rules / Quotient rules for implicit differentiation:***

For  $y$  as a function of  $x$ :  $y = f(x) \Rightarrow \begin{cases} \frac{d}{dx}(x^n y^m) = \\ \frac{d}{dx}\left(\frac{x^n}{y^m}\right) = \end{cases}$

Ex: Determine  $\frac{dy}{dx}$  of the following:

a)  $\tan(x^3 y^2) + e^{x+2y} = 3$

b)  $\sqrt{\frac{x^2+1}{y^2+3}} + x^3 - 5y = y^3$

Ex: Find an equation of the tangent line to the circle  $x^2 + y^2 = 25$  at  $x = 2$

Ex: a) Find  $\frac{dy}{dx}$  for  $x^3 + y^3 = 6xy$

b) Find the tangent to the folium of Descartes  $x^3 + y^3 = 6xy$  at the point (3,3)

c) At what points on the curve is the tangent line horizontal?

Ex: Find equations of all the tangent lines to the ellipse  $3x^2 + 4y^2 = 36$  where the slope of the tangent line  $m = -1$

Ex: Find the coordinates of the points on the graph of  $(x - 2y - 1)^2 + (x + y)^2 = 16$  where the tangent line is horizontal.

Ex: Find slope of tangent lines to the curve  $y^4 = y^2 - x^2$  at  $\left(\frac{\sqrt{3}}{4}, \frac{\sqrt{3}}{2}\right)$  and  $\left(\frac{\sqrt{3}}{4}, \frac{1}{2}\right)$