Section 3.7 Rates of Change in the Natural and Social Sciences:

Given a distance function: s(t)

Velocity function v(t)

Acceleration function: a(t)

Ex: A particle moves according to the law of motion $s = f(t)$

a) Find the velocity and acceleration at time t.
b) When the is the particle at rest?
c) When is the particle moving in the positive/negative direction?
d) Sketch the motion of the particle and then find the total distance it travel during the first 7 sec.

$s = f(t) = t^3 - 10t^2 + 25t + 2$

Ex: An objection is thrown vertically upward with a velocity of 120 ft/sec from the a platform that .is 80 ft high from the ground level, the distance function of the object: $s = f\left(t\right) = -16t^2 + 100t + 80$

a) Find the velocity after 2 seconds and after 4 seconds
b) What is the maximum height?
c) When does it hit the ground?
d) With what velocity does it hit the ground?

Ex: Suppose that the cost (in dollars) for a company to produce x pairs of new line of jeans is
$C(x) = 2000 + 3x + 0.01x^2 + 0.0002x^3$

a) Find the marginal cost function.

b) Find $C'(100)$ and explain its meaning. What does it predict?

c) Compare $C'(100)$ with the cost of producing 101^{st} item.

Section 3.8 Exponential Growth and Decay

Let $y(t)$ be the value of quantity y at time t and if the rate of change of y with respect to t is proportional to its size $y(t)$ at any time t. $\frac{dy}{dt} = ky(t)$

<u>Theorem:</u> The only solutions of the differential equation $\frac{dy}{dt} = ky(t)$ are the exponential function

$$y(t) = y(0)e^{kt}$$

Population growth:

Ex: The population of a town grows at a rate proportional to the population present at time t. The initial population of 500 increases by 15% in 10 years. What will be the population in 30 years? How fast is the population growing at t = 30 years?

Newton's Law of Cooling:

Let $T = f(t)$ be a temperature function at time t of an objection. According to Newton's Law of Cooling, the rate of cooling is proportional to the temperature difference between the object and its surroundings,

$$\frac{dT}{dt} = k(T - T_s): \text{ where } T_s : \text{surrounding temperature.}$$

So, let $y = T - T_s \Rightarrow \frac{dy}{dt} = \frac{dT}{dt} => \frac{dT}{dt} = kT$

By the previous theorem: $y(t) = y(0)e^{kt} \Rightarrow T - T_s = (y(0) - T_s)e^{kt} \Rightarrow T(t) = (y(0) - T_s)e^{kt} + T_s$

Ex: A small metal bar, whose initial temperature was $20^0 C$ is dropped into a large container of boiling water. How long will it take the bar to reach $90^0 C$ if it is known tha tits temperature increase $2^0 C$ in 1 second? How long will it take the bar to reach $98^0 C$?