

Section 3.9

Related Rates

Steps for solving related rates problems:

Step 1: Construct unknown functions.

Step 2: Find relationship of functions in step 1.

Step 3: Find out what're given and what's looking for.

Step 4: Implicit differentiate:

Ex1: A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of $2m^3 / \text{min}$, find the rate at which the water level is rising when the water is 3m deep.

Ex2: Two ships leave the same port at 1:00 P.M. The first ship heads due north, and at 2:00 P.M is observed to be three miles due north of port going at 6mph at that instant. The second ship leaves port at the same time as the first ship, heads due east, and one hour later is observed to be four miles due east of port and going at 7mph. At what rate is the distance between the two ships changing at 2:00.M.

Ex3: At noon, ship A is 5 miles west of ship B. Ship A is sailing north at speed of 7 mph and ship B is sailing north at 9 mph. How fast is the distance between the ships changing at 2:00pm?

Ex4: A man walks along a straight path at a speed of 4ft/s. A searchlight is located on the ground 20ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15 ft from the point on the path closest to the searchlight?

Ex5: A 6 – foot tall woman is walking away from a 24 – foot street lamp at a speed of 8 ft/sec. How fast is the length and the tip of her shadow is changing when she is 10 ft away from the pole?

Ex6: Gravel is being dumped from a conveyor belt at a rate of $30 \text{ ft}^3/\text{min}$, and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 10 ft high?

Ex7: A 6 – foot tall woman is walking away from a 24 – foot street lamp at a speed of 8 ft/sec. How fast is the length and the tip of her shadow is changing when she is 10 ft away from the pole?

Ex8: Two cars are approaching the same intersection along road that run at right angle to each other. Car A is traveling at 45 mph, and car B is traveling at 30 mph. If at a certain instant, A is $\frac{1}{4}$ mile from the intersection and B is $\frac{1}{2}$ mile from the intersection, find the rate at which they are approaching each other at that instant.

Ex9: Water is leaking out of an inverted conical tank at a rate of $10,000 \text{ cm}^3/\text{min}$ at the same time that water is being pumped into the tank at a constant rate. The tank has height 6 m and the diameter at the top is 4m. If the water level is rising at a rate of 20 cm/min when the height of the water is 2 m, find the rate at which water is being pumped into the tank.