Section 3.10 **Linear Approximations and Differentials**

Given a function $y = f(x)$. Determine equation of the tangent line at x = a.

Def: Linear approximation of $y = f(x)$ at x = a.

Ex: Using linear approximation to approximate the following:

a) $\sqrt[5]{35}$

b) $\sin\left(50^\circ\right)$

c) $\sqrt[3]{1003}$

Ex: Suppose that after you stuff a turkey its temperature is 50ºF and you then put it in 325ºF oven. After an hour the meat thermometer indicates that the temperature of the turkey is 93ºF and after two hours it indicates 129ºF. Predict the temperature of the turkey after three hours.

Differentials

If $y = f(x)$, where f is a differentiable function, then the differential dx is an independent variable; that is, dx can be given the value of any real number.

$y = f(x) \Rightarrow y' = \frac{dy}{dx} = f'(x) \Leftrightarrow dy = f'(x)dx$ ➔ dy is a dependent variable; it depends on the values of x and dx. If dx is given a specific value and x is taken to be some specific number in the domain of f, then numerical value of dy is determined.

Ex: Compare the values of Δy and dy if $y = f(x) = x^3 + x^2 - 2x + 1$ and x changes

a) From 2 to 2.05 and b) from 2 to 2.01

Sol: $f(2) = 9; f(2.05) = 9.717625 \Rightarrow \Delta y = f(2.05) - f(2) = 0.717625$

$dy = f'(x)dx = (3x^2 + 2x - 2)dx; x = 2; dx = \Delta x = 0.05 \Rightarrow dy = (3(2)^2 + 2(2) - 2)(0.05) = 0.7$

Let $y = f(x)$ be differentiable at x = a and suppose that $dx = \Delta x$ is an increment of x. We have two ways to describe the change in f as x changes from a to $a + \Delta x$:

$\Delta f = \Delta y = f(a + \Delta x) - f(a)$

The differential estimate: $dy = f'(a)dx = f'(a)\Delta x$

Ex: The radius of a sphere was measured and found to be 21 cm with a possible error in measurement of at most 0.05cm. What is the maximum error in using this value of the radius to compute the volume of the sphere?

$V = \frac{4}{3}\pi r^3 \Leftrightarrow dV = 4\pi r^2 dr = 4\pi(21)^2(0.05) \approx 277cm^3$