Chapter Four ***Application of Differentiation***

Section 4.1 ***Maximum and Minimum Values***

<u>Def</u>: Define "Absolute Maximum/Minimum" at x = c.
Define "Relative (local) Maximum/Minimum" at x = c.

<u>Ex</u>: Find the extrema values of the following:

a) $f(x) = x^3 - 3x^2 - 25x + 75$

b) $f(x) = 5\sin(3x)$ over $[0, 2\pi]$

<u>Def</u>: A ***<u>critical number</u>*** of a function f is a number c in the domain of f such that $f'(c) = 0$ or $f'(c) = DNE$

Ex: Find critical points of the following functions:

a) $f(x) = \frac{2}{3}x^3 + \frac{5}{2}x^2 - 3x + 4$

b) $f(x) = 6x^{2/3} - x^{5/3}$

c) $f(x) = \frac{x+1}{x^2 + x + 1}$

Extreme Value Theorem: If f(x) is continuous over a closed interval [a,b], then f(x) attains an absolute maximum value f(c) and an absolute minimum f(d) at some $c, d \in [a, b]$

Steps to find extrema:
Step 1: Find critical points over interval [a,b]
Step 2: Find y – values of f(x) at critical points and also at the boundaries x = a and x = b.
Step 3: Absolute max = highest y – value, absolute min = lowest y – value.

Fermat's Theorem: If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$
Note: it doesn't mean that in order to locate extreme values simply by testing f'(x) = 0.

Ex: Find the critical numbers of the following functions:

a) $f(x) = (2x^2 - 3)^{\frac{2}{3}}$

b) $f(x) = x^3 - 3x^2 + 1$ for $-\frac{1}{2} \le x \le 4$

c) $f(x) = x - 2\sin x;\ for\quad 0 \le x \le 2\pi$

Section 4.2 The Mean Value Theorem

Rolle's Theorem: Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval [a,b].
2. f is differentiable on the open interval (a,b).
3. f(a) = f(b)

Then there is a number c in (a,b) such that f'(c) = 0.

Ex: Verify that the function satisfies the three hypotheses of the Rolle's Theorem. Then find all number c that satisfy the conclusion of the Rolle's theorem.

a) $f(x) = x^3 - 3x^2 + 2x + 5;\ [0, 2]$

b) $f(x) = \sin 2\pi x; [-1,1]$

Ex: Prove that the following equations has at most one solution:

a) $x^3 + x - 1 = 0$

b) $x^5 + 5e^{2x} - 3 = 0$

c) $7x = 2\sin\left(3x\right) + 5$ (has exactly one solution)

The Mean Value Theorem: Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval [a,b].
2. f is differentiable on the open interval (a,b).

Then there is a number c in (a,b) such that $f'(c) = \dfrac{f(b) - f(a)}{b - a} \Leftrightarrow f(b) - f(a) = f'(c)(b - a)$

Ex: Let $f(x) = x^3 - x; a = 0, b = 2$, since f is a polynomial, then f is continuous every where, so it is certainly continuous on [0,2] and differentiable on (0,2). Therefore, by MVT, there is a number c in (0,2) such that

Ex: Suppose that $f(0) = -3$ and $f'(x) \le 5$ for all values of x. How large can $f(2)$ possibly be?

Ex: At 2:00pm a car's speedometer reads 30 mph. At 2:10pm it reads 50 mph. Show that at some time between 2:00pm and 2:10pm the acceleration is exactly 120 $miles/hr^2$

Theorem: If $f'(x)=0$ for all x in the interval (a,b), then f is constant on (a,b).

Corollary: If $f'(x) = g'(x)$ for all x in an interval (a,b) then $f - g$ is constant on (a,b); i.e. $f(x) = g(x) + \text{const}$.