

Section 4.7

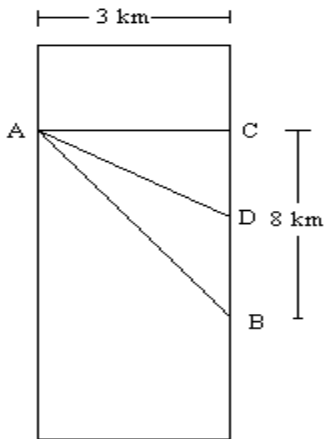
Optimization Problems

Ex: A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

Ex: What are the dimensions of an aluminum can that can hold 40 in^3 of juice and that uses the least material? Assume that the can is cylindrical, and is capped on both ends.

Ex: A box with a square base, rectangular sides, and open top is to contain 1 cubic foot of space. If the material for its base costs $\$3/\text{ft}^2$ and that for its sides costs $\$1/\text{ft}^2$, determine its dimensions so that the cost of the materials is a minimum.

Ex: A man launches his boat from point A on a bank of a straight river, 3 km wide, and wants to reach point B, 8 km downstream on the opposite bank, as quickly as possible. He could row his boat directly across the river to the Point C and then run to B, or he could row directly to B, or he could row to some point D between C and B and then run to B. If he can row 6 km/h and run 8 km/h, where should he land to reach B as soon as possible? (We assume that the speed of the water is negligible compared with the speed at which the man rows)



Ex: A gutter is made by bending a long piece of sheet metal into thirds along its length, so that the cross section is an open trapezoid. How should the bending angle θ be chosen so that the area of the cross section is as large as possible?



Ex: Find the volume of the largest cylinder that can be inscribed in the sphere of radius 5 inches.

Ex: Find the dimension of the largest cone that can be inscribed in the sphere of radius 5 inches.

Ex: Find the volume of a the largest cylinder that can be inscribed in a cone of radius 4 inches and height 10 inches.

Ex: An apple farm yields an average of 30 bushels of apples per tree when 20 trees are planted on an acre of ground. Each time 1 more tree is planted per acre, the yield decreases 1 bu. per tree due to the extra congestion. How many trees should be planted to get the highest yield? $y = (30 - x)(20 + x)$

Ex: The postal service places a limit of 84 inches on the combined length and girth (distance around) of a package to be sent parcel post. What dimensions of a rectangular box with square cross section with contain the largest volume that can be mailed?