Chapter 5 ***INTEGRALS***

Section 5.1 *Areas and Distances*

Introduce left / right hand sums of f(x) over interval [a,b]

Ex: Find R_4, L_4 of the following functions over the interval [0,1]

a) $f(x) = x^3$

b) $f(x) = \dfrac{2}{x+3}$

Note: $\sum_{i=1}^{n} c = nc; \sum_{i=1}^{n} i = \frac{n(n+1)}{2}; \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}; \sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2}\right]^2$

Ex: Find the exact area of the following functions over the interval [a,b]

a) $f(x) = 3x^2 - 4x + 7$ *over* $[0,2]$

b) $f(x) = 7x^3 - 5x + 2$ *over* $[0,3]$

Ex: Using geometry to find exact area of the following:

a) $f(x) = |3x - 6|$ *over* $[-1, 5]$

b) $f(x) = 2\sqrt{25 - x^2} + 3x$ over the interval $[0, 5]$

Section 5.2 *The Definite Integral*

Properties of the Definite Integral

a) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

b) $\int_a^a f(x)dx = 0$

c) $\int_a^b cdx = c(b-a)$

d) $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

e) $\int_a^b cf(x)dx = c\int_a^b f(x)dx$

f) $\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$ for $a \le b \le c$

g) If $f(x) \ge 0$ for $a \le x \le b$, then $\int_a^b f(x)dx \ge 0$

h) If $f(x) \ge g(x)$ for $a \le x \le b$, then $\int_a^b f(x)dx \ge \int_a^b g(x)dx$

i) If $m \le f(x) \le M$ for $a \le x \le b$, then $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$

Ex: Prove the following inequalities:

a) $\int_0^{\pi/4} \sin^3 x dx \le \int_0^{\pi/4} \sin^2 x dx$

b) $\int_1^3 \sqrt{1+x^4}\,dx \ge \frac{26}{3}$

c) $\frac{\pi}{6} \le \int_{\pi/6}^{\pi/2} \sin x dx \le \frac{\pi}{3}$

d) $2 \le \int_{-1}^{1} \sqrt{1+x^2}\, dx \le 2\sqrt{2}$