Section 5.4 ***Fundamental Theorem of Calculus (FTC)***

Let $g(x)$ be a continuous function over the interval $[a,b]$. Define $f(x)=\int_a^x g(x)\,dx \ \ for\ \ x\in[a,b]$

Fundamental Theorem Of Calculus (FTC) Part I:

If $f(x)=\int_a^x g(x)\,dx$ ==> *Then* $\dfrac{d}{dx}f(x)=f'(x)=g(x)$

Proof:

Ex: Differentiate the following functions:

a) $f(x)=\int_{0}^{x}\frac{\sqrt{3t^2-t+1}}{t^3+1}\,dt$

b) $f(x)=\int_{3}^{3x^2-5x+8}\cos\left(\frac{t^3+1}{t^2+5}\right)dt$

c) $f(x)=\int_{\cos(x)}^{e^{2x}}\sqrt{\frac{t^2+t+4}{t^4+1}}\,dt$

d) $f(x)=\sin^3\left(\int_4^{2x^3+4}\left(t^3+2t+4\right)dt\right)$

Fundamental Theorem of Calculus Part II: $\int_a^b f(x)\,dx = F(b)-F(a)\;\; where\;\; F'(x)=f(x)$

Ex: Evaluate the following:

a) $\int_0^2 \left(3x^2 - 8x + 1\right) dx$

b) $\int_{-1}^1 \left(\frac{1}{1+x^2} - 5\cos\left(2\pi x\right)\right) dx$

c) $\int_0^{\pi/3} \left(e^{-2x} - \sec^2\left(2x\right) + 4\right) dx$

d) $\int_{-1}^{3} f(x)\,dx \;\; where \;\; f(x) = \begin{cases} 1 - x^2 & if \;\; x \le 0 \\ e^{2x-1} & if \;\; x > 0 \end{cases}$

Indefinite integrals:

Ex: Integrate the following:

a) $\int \left(7x^3 - 5x + 2\right) dx$

b) $\int \frac{2\sqrt{x^3} - 5\sqrt[3]{x^2} - 7}{\sqrt{x}} dx$

c) $\int \left(e^{-2x} - 4x + \frac{3}{x\sqrt{x^2 - 1}}\right) dx$