Section 5.5 **The Substitution Rule**

So far we just solved rather than very simple integrations, but there are problems which are impossible to integrate directly such as $\int (2x-1)\sqrt{3x+7}dx$. The first technique that we discuss today is integration by substitution. The idea here is to introduce another variable (dummy variable) to make the integral simple.

The Substitution Rule: If $u = g(x)$ is differentiable function whose range is an interval I and f is continuous on I, then $\int f(g(x))g'(x)dx = \int f(u)du$

Ex: Integrate the following:

a) $\int x^3(x^4+1)^5 dx$

b) $\int x^2 e^{5x^3+1}dx$

c) $\int_0^1 (2x-1)^{20}\, dx$

d) $\int_0^1 \sqrt[5]{2x+32}\,dx$

e) $\int \frac{3x-5}{\sqrt{4x+1}}\,dx$

f) $\int x^7 \sqrt[5]{3x^4-2}\,dx$

g) $\int \left(2+\sqrt{x}\right)^{20} dx$

h) $\int \sqrt{2+\sqrt{3x+1}}\,dx$

i) $$\int \frac{x}{\sqrt{2+\sqrt{3x^2+5}}}\,dx$$

j) $$\int \sqrt{\frac{x+1}{x-1}}\,dx$$

Symmetry

Suppose f is continuous on $[-a, a]$

a) If f is even (i.e. $f(-x) = f(x)$), then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$

b) If f is odd (i.e. $f(-x) = -f(x)$), then $\int_{-a}^{a} f(x)dx = 0$

Ex: Evaluate the following:

a) $\int_{-1}^{1} \frac{\tan x}{1 + x^2 + x^4} dx$

b) $\int_{-3}^{3} \frac{x^4 \sin x}{1 + x^8} dx$