Section 2.4 The precise Definition of a Limit

Note that limit of a function f(x) is L as x approaches to a number a, means that as x gets closer to a, then f(x) gets closer to L. But in mathematics, what does it mean by "closer to a", it's very vague, we need a more precise definition of limits.

Ex: $\lim_{x \to 4}(2x-5)=3$

Def: Suppose that $f(x)$ is defined in an open interval containing the point a except possibly not at a itself). Then we say that the number L is the limit of $f(x)$ as x approaches a $\lim_{x \to a} f(x) = L$ provided the following criterion is satisfied: Given any number ε>0, there exists a corresponding number δ such that $|f(x)-L|<\varepsilon$ for all x such that $0<|x-a|<\delta$

Notation: *Given* $\forall \varepsilon > 0$ <u>*n.t.s.*</u> $\exists \delta > 0$ *such that* $|x-a|<\delta \Rightarrow |f(x)-L|<\varepsilon$

Ex: Using $\varepsilon-\delta$ definition of limit to prove the following:

a) $\lim_{x \to 4}(3x-9)=3$

b) $\lim_{x \to 4} \left(3 - \frac{1}{2}x \right) = 1$

c) $\lim_{x \to -7} \frac{x^2 + 5x - 14}{x + 7} = -9$

c) $\lim_{x \to 2}(x^2 - 7x + 5) = -5$

d) $\lim_{x \to \frac{1}{2}}\left(4x^2 - 5x + 3\right) = \frac{3}{2}$

Limits as x approaches $\pm\infty$

Note: $\lim_{x\to\pm}\frac{k}{x^n}=0$ and $\lim_{x\to\infty} r^n=0$ *iff* $|r|<1$ for any constant k and $n>0$ ➔

Limit of $\lim_{x\to\pm\infty}\frac{P(x)}{Q(x)}\Rightarrow$ Divide top and bottom by the dominant terms.

Ex: Evaluate the following:

a) $\lim_{x\to\infty}\frac{7x^3-2x+5}{4x^3+3x^2-1}$

b) $\lim_{x\to\infty}\frac{\sqrt[3]{8x^6-3x^5+4}}{10x^2-3x+4}$

c) $\lim_{x\to\infty}\frac{3^x-5^x+7^x}{7^{x+1}+2^x-5^{x-1}}$

d) $\lim_{x\to\infty} \frac{\cos(2x-\pi)}{\sqrt{7x+5}}$

e) $\lim_{x\to\infty} e^{\frac{7x^2-5}{2x^2+x-3}}$

f) $\lim_{x\to\infty} \left(\ln(3x+1)-\ln(7x+5)\right)$

g) $\lim_{x\to\infty} \left(\frac{1}{\sqrt{3x^2+4x}-\sqrt{3x^2-1}}\right)$