Section 2.5 **Continuity**

Let's look at a function is not continuous at x = a.

Def: A function f is continuous at a number a if

$$\lim_{x \to a} f(x) = f(a) \Leftrightarrow \begin{cases} \lim_{x \to a^+} f(x) = L \\ \lim_{x \to a^-} f(x) = L \\ f(a) = L \end{cases}$$

Ex: Determine if f(x) is continuous at x = a. If not, indicate f(x) is jumped / removable discontinuous at x = a.

a) $$f(x) = \begin{cases} \sqrt{3x+1} + 2; \ if \ \ x > 5 \\ 3\sin\left(\dfrac{\pi x}{10}\right) + x - 2; \ if \ x \le 5 \end{cases}; \ a = 5$$

b) $f(x)=\begin{cases} \dfrac{2x-1}{x+2} & if \;\; x>-3 \\ x^2-x-1; & if \;\; x\geq -3 \end{cases}$; $a=-3$

c) $f(x)=\begin{cases} \dfrac{2x^2+5x-3}{3x^2+8x-3} & if \;\; x\neq -3 \\ \dfrac{2}{3}; & if \;\; x=-3 \end{cases}$; $a=-3$

<u>Theorem</u>: - Any polynomial, sine, cosine, exponential function is continuous everywhere
- Any rational function is continuous on its domain.

Def: A function f is continuous from the right at a number a if $\lim_{x\to a^+} f(x) = f(a)$ and f is continuous from the left at a number a if $\lim_{x\to a^-} f(x) = f(a)$

Def: A function f is continuous on an interval if it is continuous at every number in the interval.

Theorem: If f and g are continuous at a and c is a constant, then the following functions are also continuous at a

f+g, f-g, cf, fg, f/g.

Theorem: If f is continuous at b and $\lim_{x\to a} g(x) = b$, then $\lim_{x\to a} f(g(x)) = f(b)$. In other words,

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$$

Ex: Evaluate $\lim_{x\to 1} \arcsin\left(\frac{1-\sqrt{x}}{1-x}\right) = \arcsin\left(\lim_{x\to 1} \frac{1-\sqrt{x}}{(1-\sqrt{x})(1+\sqrt{x})}\right) = \arcsin\left(\lim_{x\to 1} \frac{1}{1+\sqrt{x}}\right) = \arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Theorem: If g is continuous at a and f is continuous at g(a), then the composite function $f \circ g$ (x) is continuous at a.

Ex: Find a constant k so that the function is continuous everywhere.

$$f(x) = \begin{cases} 2x^2 + kx + 3; \ if \ \ x > 2 \\ k\sin\left(\frac{\pi x}{4}\right) - 3x \ ; \ \ if \ \ x \le 2 \end{cases}$$

The Intermediate Value Theorem (IVT): Suppose that f is continuous on the closed interval [a,b] and let N be any number between f(a) and f(b). Then there exists a number c in (a,b) such that $f(c) = N$.

Ex: Show that there is a root of the equation $4x^3 - 6x^2 + 3x - 2 = 0$ between 1 and 2.

Ex: Using IVP to show that the following equation has at least one eral solution:

a) $e^{2x} = \sin(3x) + 5$

b) $x^2 = 3\sqrt{x+1} + 2$