Section 2.7 ***Derivatives***

Given a function $y = f(x)$, and a number a is in the domain of the function f.
Find an average rate of change of the y – value over the interval [a,b]

Find an average rate o change of the y – value over the interval $\left[x, x+h\right]$

Def: The derivative of a function f at a number a, denoted by $f'(a)$ is defined as

$$f'(a) = \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} = \lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$f'(x) = \frac{df}{dx} = \frac{dy}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Ex: Use definition of derivative to differentiate the following functions:

a) $f(x) = 2x^2 - 5x + 1$

b) $f(x) = \frac{2x-1}{3-4x}$

c) $f(x) = \frac{1}{\sqrt{4x-5}}$

Interpretation of the Derivative as the Slope of a tangent.

The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.

Equation of the tangent line to $(a, f(a))$: $y - f(a) = f'(a)(x-a)$

Ex: Find an equation of the tangent line to the following function at the specified x-value.

a) $y = x^2 - 5x + 6$; x = 2.

b) $y = \sqrt{4x+5};\ at\ x = 5$

Ex: Find point(s) on the curve of $y = 3x^2 - 5x + 2$, where the slope of tangent line to the curve are 0, 3

Interpretation of the Derivative as a Rate of Change

The derivative $f'(a)$ is the instantaneous rate of change of $y = f(x)$ with respect to x when x = a.

Ex: The position of a particle is given by the equation of motion $s(t) = \dfrac{2-t}{3t+1}$ Find the instantaneous rate of change at t = 3 seconds.

Ex: A manufacturer produces bolts of a fabric with a fixed width. The cost of producing x yards of this fabric is $C = f(x)$ dollars.

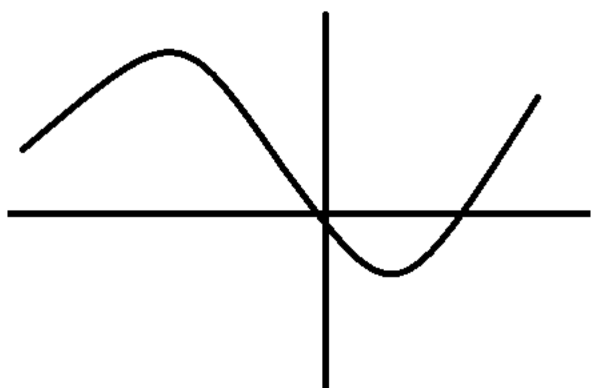
a) What is the meaning of the derivative $f'(x)$? What are its units?

b) In practical terms, what does it mean to say that $f'(1000) = 9$?

c) Which do you think is greater $f'(50)$ or $f'(500)$? What about $f'(5000)$?

Ex: The following graphs of f(x). Sketch a possible graph of their derivative functions.

a)



b)