Section 3.1 (cont.) **Derivative of Polynomials and Exponential Functions**

Given $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

1. $f(x)=k \Rightarrow f'(x)=0$

2. $f(x)=x^n \Rightarrow f'(x)=nx^{n-1}$

Proof:

Given that $x^n-a^n=(x-a)\left(x^{n-1}+x^{n-2}a+x^{n-3}a^2+\cdots+xa^{n-2}+x^{n-1}\right)$

$$f(x)=x^n \Rightarrow f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{x\to a}\frac{x^n-a^n}{x-a}=$$

$$\lim_{x\to a}\frac{(x-a)\left(x^{n-1}+x^{n-2}a+x^{n-3}a^2+\cdots+xa^{n-2}+a^{n-1}\right)}{x-a}$$

$$=\lim_{x\to a}\left[x^{n-1}+x^{n-2}a+x^{n-3}a^2+\cdots xa^{n-2}+a^{n-1}\right]=\underbrace{a^{n-1}+a^{n-1}+\cdots+a^{n-1}}_{n}=na^{n-1}$$

Ex: Differentiate the following functions:

a) $f(x)=x^{3/2}$

b) $f(x)=\frac{1}{\sqrt[13]{x^5}}$

Ex: Prove the following:

a) $g(x) = kf(x) => g'(x) = kf'(x)$

b) $g(x) = f(x) + h(x) \Rightarrow g'(x) = f'(x) + h'(x)$

Ex: Differentiate the following functions:

a) $f\left(x\right)=7x^{3}-5x^{2}+4x-13$

b) $f\left(x\right)=\left(7x^{2}-5\right)^{2}$

c) $f\left(x\right)=\dfrac{7\sqrt{x^{5}}-2\sqrt[3]{x^{4}}+20}{10\sqrt{x^{3}}}$

Given: $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$ ➔ Prove that $f(x) = e^x$ => $f'(x) = e^x$

Ex: Find equation of tangent line to the function: $f(x) = 3e^x + 5\sqrt{x} - 9\sqrt[3]{x^5} + 2$ at x=1.

Ex: At what point on the curve $y = e^x$ is the tangent line parallel to the line $y = 5x - 1$

Ex: Find a parabola with equation $y = ax^2 + bx$ whose tangent line at (1,1) has equation $y = 3x - 2$