Section 3.6 ***The Chain Rule***

How do we differentiate, say $\left(x^2 - x + 1\right)^{100}$, even though the function is quite simple, but we don't want to expand it and then find the derivative term by term ➔ chain rule?

The Chain Rule: $f(x) = (u \circ v)(x) \Rightarrow f'(x) = u'(v(x))v'(x) = \frac{du}{dx}(v(x))\frac{dv}{dx}$

Ex: Find the derivative of the following functions:

a) $f(x) = \left(2x^2 - 5x + 1\right)^{50}$

b) $f(x) = \sqrt[3]{5x^2 \sin x}$

c) $f(x)=\sin^3\left(\dfrac{x^3+2}{3x^4-1}\right)$

d) $f(x)=\tan^3\left(\sin\left(\sqrt{3x^2+1}\right)\right)$

e) $f(x)=\sqrt[5]{\dfrac{x^5+2}{\cot(3x+1)}}$

Ex: Find an equation of the tangent line to the curve at the given point.

$f(x) = 2x\sin^3(x+1); x = -1$

Ex: Find second derivative of $f(x) = (2x-3)\sin(2x)$

Ex: Find $\frac{d^{100}}{dx^{100}}\left(\sin\left(2x\right)\right)$

Section 3.7 ***Implicit Differentiation***

So far all the differentiation that we have discussed, have the same type of problems, we call those are explicit, because y or f(x) have been explicitly expressed as a function of x. How about equations that we can not express y explicitly as a function of x, such as equation of a circle…

Ex: Differentiate the following equations:

a) $x^2 - 5xy + y^3 = 1$

b) $x^3 + \sin y - 5x + 3y = 0$

c) $\sin(x + y) = y^2 \cos x$

Ex: Find an equation of the tangent line to the circle $x^2 + y^2 = 25$ at $x = 2$

Ex: a) Find y' if $x^3 + y^3 = 6xy$

b) Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point (3,3)

c) At what points on the curve is the tangent line horizontal?

Ex: Find equations of all the tangent lines to the ellipse $3x^2 + 4y^2 = 36$ where the slope of the tangent line is -1.

Ex: Find the coordinates of the points on the graph of $(x - 2y - 1)^2 + (x + y)^2 = 16$ where the tangent line is horizontal.

Orthogonal Trajectories:

Def: Two curves are called orthogonal if at each point of intersection their tangent lines are perpendicular.

Ex: Consider the graph of all ordered pairs (x, y) that satisfy the equation $\frac{x}{x^2 + y^2} = 4$ and the graph of the ordered pairs that satisfy the equation $\frac{y}{x^2 + y^2} = -2$. Show that these two graphs is orthogonal; that is, any point where the graphs intersect, the tangents to the graphs are at right angles to each other.

Section 3.8 ***Higher Derivatives***

Notation: $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2 y}{dx^2}; f''(x) = D^2 f(x); f^{(n)}(x) = D^n f(x)$

Ex: Given the function $f(x) = x^3 \cos(2x - 1)$, find and interpret f''(x).

Another example here to find higher derivatives, also mention derivative of polynomial of degree n, then derivative n+1st of that polynomial would give the zero function. Derivative of exponential function e^x is always that function regardless of the degree of differentiation.

- Mention that if s(t) is a position function then s'(t) is the velocity, and s''(t) is the acceleration.

Ex: The position of a particle is given by the equation $s(t) = t^3 - 6t^2 + 9t$ where t is measured in seconds and s in meters.

a) Find the acceleration at time t. What is the acceleration after 4s?
b) Graph the position, velocity, and acceleration functions for $0 \le t \le 5$.
c) When is the particle speeding up? When is it slowing down?

Ex: If $f(x) = \frac{1}{x} \Rightarrow$ find $f^{(n)}(x)$

Ex: Find y'' if $x^4 + y^4 = 16$

Ex: Find $D^{75} \sin x$