Vector Differential Equations: Defective Coefficient Matrix

Def: Let A be an n by n matrix, and it has m eigenvectors where m < n. Then A is called a defective matrix.

Ex: Solve $\dfrac{\overrightarrow{dx}}{dt} = A\overrightarrow{x}$ where $a)$ $A = \begin{bmatrix} 6 & -8 \\ 2 & -2 \end{bmatrix}$

b) $A=\begin{bmatrix} 0 & 1 \\ -9 & 6 \end{bmatrix}$

What if Eigenvalue of multiplicity three: Say a matrix A has eigenvalue of multiplicity 3 with eigenvector K.

Then we have: $\begin{cases} (A-\lambda I)\overrightarrow{K} = \overrightarrow{0} \\ (A-\lambda I)\overrightarrow{P} = \overrightarrow{K} \\ (A-\lambda I)\overrightarrow{Q} = \overrightarrow{P} \end{cases}$

Then the general solution:

$\overrightarrow{x(t)} = C_1\overrightarrow{x_1} + C_2\overrightarrow{x_2} + C_3\overrightarrow{x_3}$

$where\ \overrightarrow{x_1} = e^{\lambda t}\overrightarrow{K};\ \ \overrightarrow{x_2} = e^{\lambda t}\left(\overrightarrow{P} + t\overrightarrow{K}\right)\ and\ \overrightarrow{x_3} = e^{\lambda t}\left(\overrightarrow{Q} + t\overrightarrow{P} + \frac{t^2}{2!}\overrightarrow{K}\right)$

That means: the solution: $\overrightarrow{x(t)} = C_1 e^{\lambda t}\overrightarrow{K} + C_2 e^{\lambda t}\left(\overrightarrow{P} + t\overrightarrow{K}\right) + C_3 e^{\lambda t}\left(\overrightarrow{Q} + t\overrightarrow{P} + \frac{t^2}{2!}\overrightarrow{K}\right)$

d) $$A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & -1 & 1 \end{bmatrix}$$

e) $$A = \begin{bmatrix} 2 & 1 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$$