The Laplace Transform and some Elementary Applications

Section 7.1 *The definition of the Laplace Transform*

Review improper integrals:

Def: Let f be a function defined on the interval $[0,\infty)$. The Laplace transform of f is the function F(s) defined by

$$F(s)=\int_0^\infty e^{-st}f(t)dt$$

provided that the improper integral converges. We will usually denote the Laplace transform of f by $L[f]$.

Note that $\int_0^\infty e^{-st}f(t)dt=\lim_{b\to\infty}\int_0^b e^{-st}f(t)dt$ and convergence means

$$\int_0^\infty e^{-st}f(t)dt=\lim_{b\to\infty}\int_0^b e^{-st}f(t)dt=L<\infty$$

Ex: Determine the Laplace transforms of the following functions:

a) $f(t) = 1$

b) $f(t) = t$

c) $f(t) = t^n$

d) $f(t) = e^{at}$

e) $f\left(t\right)=\sin(bt)$

f) $f\left(t\right)=\cos\left(bt\right)$

h) $L(\sinh(bt)) = \dfrac{b}{s^2 - b^2}$; $L(\cosh(bt)) = \dfrac{s}{s^2 - b^2}$

g) $$f(t) = \begin{cases} 2; & 0 < t < 5 \\ 0; & 5 < t < 10 \\ e^{4t}; & t > 10 \end{cases}$$

h) $L(f(t))$ *where* $f(t)=\begin{cases}0; & 0\le t<3\\ 2; & t\ge 3\end{cases}$

Theorem: Let f and g be functions whose Laplace transform exist for $s>\alpha$ and let c be a constant. Then

1. $L[f+g]=L[f]+L[g]$
2. $L[cf]=cL[f]$

Ex: Determine the Laplace function for $f(t) = 11 + 5e^{4t} - 6\sin 2t$