Section 7.3 ***The First Shifting Theorem***

- Evaluating $L(e^{5t}t^3)$ *and* $L(e^{-2t}\cos 4t)$ is straightforward provided that we known (and we do) $L(t^3)$ *and* $L(\cos 4t)$. In general, if we know the Laplace transform of a function f $L(f)=F(s)$, it is possible to compute the Laplace transform of an exponential multiple of f (t)- that is, possible to compute the Laplace transform of an exponential multiple of f, that is, $L(e^{at}f(t))$- with no additional effort other than *translating*, or *shifting*, the transform $F(s)$ *to* $F(s-a)$. This result is known as the **first translation theorem** or **fist shifting theorem.**

First Translation Theorem: (Translation on the s – axis)

If $L(f(t))=F(s)$ and a is any real number, then $L(e^{at}f(t))=F(s-a)$

Proof: $L(e^{at}f(t))=\int_0^\infty e^{-st}e^{at}f(t)dt=\int_0^\infty e^{-(s-a)t}f(t)dt=F(s-a)$

In general:

1. $L\left(e^{at}t^n\right)=\dfrac{n!}{(s-a)^{n+1}};\Rightarrow L^{-1}\left(\dfrac{n!}{(s-a)^{n+1}}\right)=e^{at}t^n$

2. $L\left(e^{at}\cos bt\right)=\dfrac{s-a}{(s-a)^2+b^2};\Rightarrow L^{-1}\left(\dfrac{s-a}{(s-a)^2+b^2}\right)=e^{at}\cos(bt)$

3. $L\left(e^{at}\sin bt\right)=\dfrac{b}{(s-a)^2+b^2};\Rightarrow L^{-1}\left(\dfrac{b}{(s-a)^2+b^2}\right)=e^{at}\sin(bt)$

4. $L\left(e^{at}\sinh(bt)\right)=\dfrac{b}{(s-a)^2-b^2};\Rightarrow L^{-1}\left(\dfrac{b}{(s-a)^2-b^2}\right)=e^{at}\sinh(bt)$

5. $L\left(e^{at}\cosh(bt)\right)=\dfrac{s-a}{(s-a)^2-b^2};\Rightarrow L^{-1}\left(\dfrac{s-a}{(s-a)^2-b^2}\right)=e^{at}\cosh(bt)$

Note: If we consider s a real variable, then the graph of $F(s-a)$ is the graph $F(s)$ is shifted a units to the right.

Ex: Evaluate the Laplace Transform of the following functions:

a) $f(t)=t^3e^{4t}$

b) $f(t)=e^{-2t}\cos(4t)$

c) $f(t)=e^{-7t}\sin(3t)+e^{3t}\left(t^3-2t+5\right)+3$

d) $f(t)=e^{5t+2}\cosh(7t)+7e^{-3t}\sinh(2t)$

Ex: Determine $L^{-1}(F(s))$ for the following:

a) $F(s)=\dfrac{s}{s^2+6s+11}$

b) $F(s)=\dfrac{1}{(s-1)^3}+\dfrac{1}{s^2+2s-8}$

c) $F(s)=\dfrac{2s+5}{(s-3)^2}$

d) $$L^{-1}\left(\frac{s/2+5/3}{s^2+4s+6}\right)=$$

e) $$F(s)=\frac{s^2-17s+32}{s^3-6s^2+16s}$$

f) $$F(s)=\frac{3s^2+14s+20}{s^2+4s+9}$$

Ex: Using Laplace Transform to solve the following:

a) $y''+4y'+6y=1+e^{-t};\ y(0)=0;\ y'(0)=0$

b) $y''+2y'-3y = 26e^{2t}\cos t;\ y(0)=1;\ y'(0)=0$