Oscillation Cont.

$$F = ma = m\frac{d^2x}{dt^2} = F_{gravity} + F_{spring} + F_{damping} + F_{external}$$

$$m\frac{d^2x}{dt^2} = mg - (kx + kL_0) - c\frac{dx}{dt} + F(t) = (mg - kL_0) - kx - c\frac{dx}{dt} + F(t)$$

$$Note: Equilibrium\ \ position: \ mg = kL_0 \Rightarrow mg - kL_0 = 0$$

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = F(t)\begin{cases} x(0) = x_0 \\ x'(0) = v_0 \end{cases}$$

Harmonic motion: No damping force, and external force➔

$$m\frac{d^2x}{dt^2} + kx = 0 => \frac{d^2x}{dt^2} + \frac{k}{m}x = 0 ==> \frac{d^2x}{dt^2} + \varpi^2 x = 0 \ \ where\ \varpi = \sqrt{\frac{k}{m}}$$

⇨ Solution:

$$x(t) = C_1\cos(\varpi t) + C_2\sin(\varpi t) = A\cos(\varpi t - \delta)$$

$$x(0) = A\cos(-\delta) = A\cos(\delta) = x_0 \Rightarrow A^2\cos^2(\delta) = {x_0}^2$$

⇨ $$x'(t) = -\varpi A\sin(\varpi t - \delta) \Rightarrow x'(0) = -\varpi A\sin(-\delta) = \varpi A\sin(\delta) = v_0 => A^2\sin^2\delta = \frac{{v_0}^2}{\varpi^2}$$

$$==> A^2 = {x_0}^2 + \frac{{v_0}^2}{\varpi^2} => A = \sqrt{{x_0}^2 + \frac{{v_0}^2}{\varpi^2}} \ \ and\ \ \tan(\delta) = \frac{v_0}{\varpi x_0} => \delta = \tan^{-1}\left(\frac{v_0}{\varpi x_0}\right)$$

Ex: An object with a weight of 25 lb that stretches a spring 10 inches beyond its natural length. Initially, the object is pulled 1 ft below its equilibrium position and release with an upward push of initial velocity of 2 ft/sec. Assume that there's no damping force and no external force. Describe the motion of the mass as a function to time t.

Sol:

Ex: A mass – spring obeys the following IVP:

i) Describe the movement of the mass – spring system as function of time t (in seconds).

ii) Determine when (if any) the mass crosses the equilibrium position and how fast.

iii) Determine the time when the amplitude of the spring is less than 0.001.

a) $8y''+10y'-3y=0;\ \ y(0)=1 \text{ and } y'(0)=-2$

b) $9y''+24y'+16y=0;\ y(0)=1;\ y'(0)=-2$

c) $8y''+10y'-3y=0;\ y(0)=-1,\ y'(0)=0$

Ex: A force of 32 lb that can stretch a spring 8/3 ft beyond its natural length and the object is placed in a liquid that offers a resistant force of numerically 7 times its instantaneous speed. Initially pulled 1 ft below its equilibrium position and release with an upward push of initial velocity of 20 ft/sec.

a) Describe the motion of the mass as a function to time t.

b) Does the mass ever cross the equilibrium position, why or why not ?

c) Determine the time when the distance between the object and its equilibrium position is less than 0.01 ft.