Section 2.2 ***Separable Equations***

Def: A first – order DE of the form: $y' = \frac{dy}{dx} = g(x)h(y)$ is said to be separable

Ex: Solve the following DE:

a) $\frac{dy}{dx} = \frac{e^{2x-y+1}}{e^{2y-x-3}}$

b) $\frac{dP}{dt} = p^2 - p - 20$

c) $y - x\frac{dy}{dx} = 3 - 2x^2\frac{dy}{dx}$

d) $\frac{dy}{dx} = \frac{x^2 y - 32}{16 - x^2} + 2$

e) $\frac{dy}{dx} = \frac{x \tan^{-1} x}{y}; \; y(0) = 3$

Ex: A tank contains 600 gal of water in which there is dissolved 4lbs of salt. A solution containing ½ lb of salt flows into the tank at the rate of 5gal/min, and the well-stirred mixture flows out at the same rate of 5 gal/min. Determine the concentration of salt in the tank after 1 hour.

Ex: Using differential equation compute the monthly payment of a mortgage loan of $650,000 at a fixed rate of 5.5% per year compounded continuous for 30 yrs.

Section 2.2 ***First-Order Linear DE***

Def: A DE that can be written in the form $a(x)\frac{dy}{dx}+b(x)y=r(x)$ where $a(x),b(x)$ and $r(x)$ are functions defined on an interval (α,β) is called a first-order linear DE.

If $a(x)\neq 0 \Rightarrow \frac{dy}{dx}+p(x)y=q(x)$

Ex: Solve the following equations:

a) $\frac{1}{x}\frac{dy}{dx}-\frac{2y}{x^2}=x\cos x;\quad y\left(\frac{\pi}{2}\right)=3$

b) $\cos x \dfrac{dy}{dx} + y \sin x = 2x \cos^2 x; \quad y\left(\dfrac{\pi}{4}\right) = \dfrac{-15\sqrt{2}\pi^2}{32}$

c) $\dfrac{dy}{dx} + \dfrac{3x}{3x^2+1} y = \dfrac{xe^{4x^2-1}}{\sqrt{3x^2+1}}$

d) $x\frac{dy}{dx} - 2y = x^3 \cos(5x)$

e) $x\frac{dy}{dx} + 3y = \frac{1}{x^2}\sin(3x)$

f) $y' - y = f(x), \quad y(0) = 0$ where $f(x) = \begin{cases} 1, & \text{if } x < 1 \\ 2 - x, & \text{if } x \geq 1 \end{cases}$

g) $\frac{dy}{dx} - \frac{3x}{3x^2+1}y = \frac{x\sqrt{3x^2+1}}{e^{x^2}}$

h) $\frac{dy}{dx} - y = f(x);\ where\ f(x) = \begin{cases} 1;\ if\ x < 1 \\ 2-x;\ if\ x \geq 1 \end{cases};\ y(0) = 0$