***Worksheet #3*** ***Math 180*** ***Name:***

1. Solve the following problems:
   a) A cardboard box is to be made from a rectangular sheet of cardboard by cutting out squares from the four corners of the cardboard and then folding up what is left to make the box. The box has a bottom and sides but no top. The cardboard sheet is 80 cm by 30 cm. What dimensions produce the box with largest volume?
   b) A farmer wants to make a rectangular field with a total area of 2000 square meters. It is surrounded by a fence. It is divided into 4 equal areas by fences. What is the shortest total length of fence this can be done with?
   c) A commuter railway has 800 passengers per day and charges each one $2. For each 2 cents that the fare is increased, 5 fewer people will go by train. What is the greatest profit that can be earned?
   d) A cylindrical metal can is to have no lid. It is to have volume $8\pi$ $in^3$. What height minimizes the amount of metal used?
   e) A university is trying to determine what price to charge for football tickets. At a price $6 per ticket it averages 70,000 per game. For every increase of $1 it loses 10,000 people from the average. Every person at the game spends an average of $1.50 on concessions. What price per ticket should be charged to maximize revenue? How many people will attend at the price?
   f) A Norman window is a rectangle with a semi-circle on top. Suppose the perimeter of a particular Norman window is to be 24 ft. What should be its dimensions so the maximum amount of light will be allowed to enter through the window?
   g) Determine the largest volume cone that can be inscribed in a sphere of radius r.
   h) Determine the largest cylinder that can be inscribed in a cone with radius r and height h.
   i) A flower bed is planted in the form of a circular sector. Find the central angle and radius if it is to cover 169 square feet, and its perimeter is to be a minimum.
   j) A straight two – lane highway intersects a straight interstate at a right angle. A car exits the interstate and starts moving away from it on the two – lane high way at 50 mph. At the same instant, another car, moving at 75 mph on the interstate, is approaching the same intersection, but is still 10 miles from it. When will their distance be a minimum and what will this distance be?

2. Analyze the graph of the following functions:

   a) $f(x) = x^{2/3} + 2$ b) $f(x) = 6 - (x-3)^{3/5}$ c) $f(x) = x^{2/3}\left(\frac{2}{3} - x\right)$

3. Find function f(x) with the following properties.
   a) $f''(x) = -6x,\ f(1) = 3,\ f(2) = 5$
   b) $f'''(x) = -12,\ f(0) = 1,\ f(1) = 6,\ f(-1) = 4$

4. Solve the following problems:
   a) A ball is dropped from a platform 19.6 m high. Find its position function in term of time t.
   b) A ball is thrown upward with an initial velocity of 14.7 m/s from the edge of a platform 19.6 m above ground level. Find the position function and calculate the displacements of the ball at time t = 3s, and t = 3.5s.
   c) A car braked with a constant deceleration of 16 ft/s$^2$ , producing skid marks measuring 200 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

5. Given function $f(x) = e^{x^2}$ and $g(x) = \ln x$
   a) Evaluate $R_{10}$ for both f(x) and g(x) over the interval $[1,2]$.
   b) Set up a limit of Riemann sum for both functions f(x) and g(x) using the midpoint of each subinterval.

6. Evaluate the following integrals by interpreting their geometry.
   a) $\int_0^3 (5x+3)dx$ b) $\int_0^5 \left(\sqrt{25 - x^2} - 1\right)dx$ c) $\int_0^5 |3x - 2|dx$

7. Prove the Fundamental Theorem of Calculus (part I and part II)

7. Find the derivative of the following functions.

a) $f(x) = \int_{3}^{x^2-2x+5} \frac{2\sin t}{1+t^4}\,dt$

b) $f(x) = \int_{2x-5}^{\frac{5-7x}{x^2-3}} \left( t\cos^2\left(\frac{3t+1}{t^2+1}\right)\right) dt$

8. Find the interval on which the curve $y = \int_0^x \frac{dt}{1+t+t^2}$ is concave upward.

9. Use definition to determine the exact area under the curve over the indicated interval.

a) $f(x) = 2x^3 - x + 1 \ \ over \ \ [-2,3]$

b) $f(x) = 5x^2 + 2x - 4 \ \ over \ \ [-1,2]$

10. Evaluate the following limits:

a) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$

b) $\lim_{x\to\infty} \frac{\sqrt{x^2+2}}{\sqrt{2x^2+1}}$

c) $\lim_{x\to 0^+} \frac{\cot x}{\cot 2x}$

d) $\lim_{x\to\infty} x\ln\left(1+\frac{a}{x}\right); a > 0$

e) $\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$

f) $\lim_{x\to 0}\left(\frac{1}{x^2} - \frac{1}{x\tan x}\right)$

g) $\lim_{x\to 0}\left(e^x + x\right)^{1/x}$

h) $\lim_{x\to 0}\left(1+ax\right)^{b/x}$

i) $\lim_{x\to 0}\left(\cos ax\right)^{b/x^2}$

j) $\lim_{x\to\infty} \tanh x$

k) $\lim_{x\to -\infty} \csc hx$

l) $\lim_{x\to 0^-} \coth x$

n) $\lim_{x\to\infty}\left(\frac{2x-3}{2x+5}\right)^{2x+1}$

o) $\lim_{x\to 0} \frac{\ln x}{1/x}$

p) $\lim_{x\to 0^+} \frac{\arcsin(1-x)}{\sqrt{2x-x^2}}$

11. Evaluate the following integrals:

a) $\int x^7\sqrt{5x^4+1}\,dx$

b) $\int \frac{x-1}{\sqrt{3x+2}}\,dx$

c) $\int \cos^3(5x)\sin^3(5x)\,dx$

d) $\int \sqrt{\cos(7x)}\,\sin^3(7x)\,dx$

e) $\int xe^{3x^2-5}\,dx$

f) $\int \frac{\ln(7x)}{x}\,dx$

g) $\int \sec^4(5x)\,dx$

h) $\int \frac{\cos(3x)dx}{\sin(3x)\sqrt{\sin^2(3x)-1}}$

a) $\int \frac{7}{16+x^2}\,dx$

b) $\int \frac{1}{x^2-4x+7}\,dx$

c) $\int \frac{3}{2\sqrt{x}(1+x)}\,dx$

d) $\int \frac{x+5}{\sqrt{6x-x^2}}\,dx$

e) $\int_{\pi/2}^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$

f) $\int_0^{1/\sqrt{2}} \frac{\arccos x}{\sqrt{1-x^2}}\,dx$

g) $\int \frac{2}{\sqrt{4x-x^2}}\,dx$

h) $\int_{-\pi/2}^{\pi/2} \frac{x^2\sin x}{1+x^6}\,dx$

i) $\int \frac{2x-5}{x^2+2x+2}\,dx$

j) $\int \frac{dx}{(x-1)\sqrt{x^2-2x}}$

k) $\int \frac{xdx}{\sqrt{9+8x^2-x^4}}$

l) $\int \frac{dx}{2\sqrt{3-x}\sqrt{x+1}} \quad let \ \ u = \sqrt{x+1}$

m) $\int \frac{e^{1/x}}{x^2}\,dx$

n) $\int \frac{x}{\sqrt{x-1}}\,dx$

o) $\displaystyle\int \frac{dx}{x\sqrt{7x^2-4}}$

r) $\displaystyle\int \sec^3(3x)\tan^2(3x)dx$

s) $\displaystyle\int \frac{dx}{\sqrt{e^{2x}-1}}$

t) $\displaystyle\int \frac{1}{\sqrt{x}\sqrt{1+x}}dx$

u) $\displaystyle\int \frac{\sqrt[7]{\ln(5x)}}{x}dx$

v) $\displaystyle\int \frac{dx}{x(\ln 7x)^5}$

w) $\displaystyle\int_1^e \frac{dx}{x\sqrt{1-(\ln x)^2}}$

x) $\displaystyle\int \frac{dx}{x\left[1+(\ln x)^2\right]}$

y) $\displaystyle\int_1^3 \frac{dx}{2\sqrt{x}(1+x)}$

z) $\displaystyle\int \frac{e^x+e^{-x}}{e^x-e^{-x}}dx$