1. Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the indicated line.
   a) \( y = \frac{1}{1 + x}; \quad y = 0, \quad x = 0, \quad x = 3 \) about \( y = 4 \).
   b) \( y = \sec x, \quad y = 0, \quad 0 \leq x \leq \frac{\pi}{3} \) about the line \( y = 4 \).
   c) \( xy = 6, \quad y = 2, \quad y = 6, \quad x = 6 \) about the line \( x = 6 \).
   d) \( y = e^{x/2}; \quad y = 0, \quad x = 0, \quad x = 4 \) about the \( x \)-axis.
   e) \( y = x^2 + 1 \quad y = -x^2 + 2x + 5, \quad x = 0, \quad x = 3 \) about the \( x \)-axis.
   f) \( (y - 2)^2 = 4 - x; \quad x = 0 \) about the \( x \)-axis.
   g) \( x^2 + y^2 = 4 \) about \( x = 3 \).
   h) \( y = 2\sin x \quad y = -\sin x; \quad 0 \leq x \leq \pi \) about the \( y \)-axis.

2. Solve the following application problems:
   a) A 10-ft chain weighs 25 lb and hangs from a ceiling. Find the work done in lifting the lower end of the chain to the ceiling so that it’s level with the upper end.
   b) A tank in the shape of a sphere with radius 10 m is full of gasoline, with density 680 kg/m\(^3\). Calculate the work required to empty the tank through a hole at the top.
   c) A tank in the shape of a right circular cylinder with radius 3 m and height 10 m is placed so that its axis is horizontal. If the tank is full of gasoline, with density 680 kg/m\(^3\), calculate the work required to empty the tank through a hole at the top of one of the circular ends.
   d) Calculate the work done in stretching a spring 0.30 m from its nature length. The force exerted by the spring when it has been stretched 0.05 m has magnitude 25N.
   e) Calculate the work done in compressing a spring 0.20 m from its nature length. The force exerted by the spring when it has been stretched 0.05 m has magnitude 30N.
   f) A rectangular swimming pool has dimensions width of 10 m, length of 50 m. Its depth is constant along lines parallel to the short sides and varies linearly from \( h = 1 \) m to \( H = 3 \) m along the long side. Assuming the pool is full, calculate the work required to empty the pool.

4. Find arc-length of the following functions
   a) \( f(x) = x^{2/3} \) from \( 1 \) to \( 8, 4 \) \( \) \( \)
   b) \( f(x) = \frac{x^2}{4} - \frac{\ln x}{2} \) from \( x = 1 \) to \( x = 2 \).
   c) \( f(x) = \ln \frac{e^x + 1}{e^x - 1} \) from \( x = 1 \) to \( x = 2 \).
   d) \( x = \frac{y^3}{3} + \frac{1}{4y} \) from \( x = 0 \) to \( x = \frac{1}{2} \).

5. Find the surface area of the following
   a) \( y = \frac{x^2}{4} - \frac{\ln x}{2} \) rotates about the \( y \)-axis from \( x = 1 \) to \( x = e \).
   b) \( y = \ln x; \) rotates about the \( y \)-axis from \( (1, 0) \) to \( (e, 1) \).
   c) \( x = \frac{1}{3}(y^2 + 2)^{1/2}; \) rotate about the \( x \)-axis for \( y = 1 \) to \( y = 2 \).

6. Solve the following problems:
   a) A dam has a square gate with side 1 m long and one diagonal vertical. The highest point of the gate is 3 m below the water surface. Find the force on the gate.
   b) A trough is 6 m long and 1 m high. Vertical cross sections are equilateral triangles with the top side horizontal. Find the force on one end if the trough is filled with water. Also find the force on one side of the trough.
   c) A dam has a vertical gate that is an isosceles trapezoid with upper base 2 m, lower base 3 m and height 1 m. Find the force on the gate if the upper base is 3 m below the surface.
7. Evaluate the following limits:
   a) \( \lim_{x \to 0} \frac{1 - \cos x}{x^2} \)
   b) \( \lim_{x \to 0} \frac{\sqrt{x^2 + 2}}{2x^2 + 1} \)
   c) \( \lim_{x \to 0} \frac{\cot x}{\cot 2x} \)
   d) \( \lim_{x \to \infty} \left(1 + \frac{a}{x}\right)^x \), \( a > 0 \)
   e) \( \lim_{x \to 0} \left( \frac{x}{x - 1} - \frac{1}{\ln x} \right) \)
   f) \( \lim_{x \to 0} \left( \frac{1}{x^2} - \frac{1}{x \tan x} \right) \)
   g) \( \lim_{x \to 0} \left( e^x + x \right)^{1/x} \)
   h) \( \lim_{x \to 0} (1 + ax)^{b/x} \)
   i) \( \lim_{x \to 0} (\cos ax)^{b/x^2} \)

8. Evaluate the following integrals:
   a) \( \int x^2 \cos(3x) \, dx \)
   b) \( \int x^3 e^{5x} \, dx \)
   c) \( \int x^5 \ln(3x) \, dx \)
   d) \( \int \frac{\sqrt{1 + \ln x}}{x \ln x} \, dx \)
   e) \( \int \frac{1 - e^x}{1 + e^x} \, dx \)
   f) \( \int \frac{dx}{x^3 - 5x^2 + 6x} \)
   g) \( \int x \sin^{-1} x \, dx \)
   h) \( \int_0^1 (1 + \sqrt{x})^{1/2} \, dx \)
   i) \( \int \frac{dx}{\sqrt{4x^3 - 4x - 3}} \)
   j) \( \int \frac{dx}{\sqrt{x + \sqrt{x}}} \)
   k) \( \int \frac{dx}{(5 - 4x - x^2)^{5/2}} \)
   l) \( \int \cos^2 x \sin 2x \, dx \)
   n) \( \int \tan^2 x \sec x \, dx \)
   o) \( \int \csc^4 x \cot^6 x \, dx \)
   p) \( \int \sin^3 x \cos^5 x \, dx \)

9. Evaluate the following improper integrals:
   a) \( \int_1^\infty \frac{dx}{1 + 5x^2} \)
   b) \( \int_1^\infty \frac{dx}{\sqrt{3x - 1}} \)
   c) \( \int_3^6 \frac{dx}{(x + 1)^{4/3}} \)
   d) \( \int_{-\infty}^\infty \frac{x^2}{9 + x^6} \, dx \)
   e) \( \int_0^\infty \cos^2 (2x - 1) \, dx \)
   f) \( \int_0^2 x^2 \ln x \, dx \)

   g) Find the area of the region bounded by the following curve:
      i. \( y = \frac{1}{(x - 1)^{1/3}} \), \( y = 0 \) between \( x = 0 \) and \( x = 2 \)
      ii. \( y = \frac{1}{\sqrt{x - 1}} \), \( y = 0 \), right of \( x = 5 \)

10. Solve the following differential equations:
    a) \( \frac{dy}{dx} = \frac{5 + \sqrt{2x}}{1 - \sqrt{3y}} \)
    b) \( y' = t \ln(y^{2t}) + t^2 \)
    c) \( \frac{dy}{dt} = \frac{2t}{1 + 2y} \), \( y(2) = 0 \)
    d) \( \frac{dy}{dx} = -\frac{1 + y^2}{1 + x^2} \), \( y(0) = -1 \)
    e) \( x \frac{dy}{dx} + 2y = x^3 \), \( x > 0 \); \( y(1) = 0 \)
    f) \( x \frac{dy}{dx} - \frac{y}{x + 1} = x; \, x > 0 \); \( y(1) = 0 \)

11. Solve the following application problems:
    a) You have an outstanding balance of $2400 on your credit card and decide to pay it off at the rate of $100 per month.
      i. Let \( b(t) \) be the balance owed at time \( t \). Find an expression for \( db/dt \)
      ii. If the interest rate is 18 percent annually, compounded continuously, how long will it take you to eliminate the balance, assuming that you make no additional charge on the card?
      iii. If you pay only $50 per month, does it take twice as long to pay back the balance? What if you pay $200 per month?
      iv. Suppose you have an opportunity to transfer the balance to a card that charges 14 percent annual interest, compounded continuously. How long will it take you to pay off the balance at $100 per month?
    b) Suppose alcohol is introduced into a 2-liter (L) beaker, which initially contains pure water, at the rate of 0.1 L/min. The well-stirred mixture is removed at the same rate.
i. How long does it take for the concentration of alcohol to reach 50%? 75%? 87.5%?
ii. Suppose the current concentration of water in the beaker is c and we ask how long it takes before the concentration is cut in half. Is this time interval the same, regardless of c?
c) You have a mortgage loan of $500,000 at interest rate 5.65% per year.
i. Set up a differential equation for the balance \( B(t) \) where \( t \) is time in years.
ii. Determine your monthly payment for a 30 year-loan.
iii. Determine the interest you pay after 30 years.
iv. Determine the balance of your loan after 5 years.
v. Set up another differential equation for the new balance \( B(t) \) if you want to refinance the loan after 5 years with a new interest of 5.0% for 15 years, then determine your monthly payment and interest for this new loan.

12. Find \( \frac{dy}{dx} \) and \( \frac{d^2y}{dx^2} \) of the following parametric equations.
   a) \( x = t - e^t; y = t + e^{-t} \)
   b) \( x = 2\sin t; y = 3\cos t \)

13. Find the points on the curve where the tangent is horizontal or vertical.
   a) \( x = 2t^3 + 3t^2 - 12t; y = 2t^3 + 3t^2 + 1 \)
   b) \( x = 2\cos t; y = \sin 2t \)

14. Sketch the graph of the following polar equations:
   a) \( r = 2/\theta^2 \)
   b) \( r = 3 + 2\sin \theta \)
   c) \( r = 3 - 7\cos \theta \)
   d) \( r^2 = \sin(2\theta) \)
   e) \( r = 5\cos(2\theta) \)
   f) \( r = 2 - 5\sin(2\theta) \)

15. Find the area of the following regions:
   a) Inside the cardioid \( r = a(1 + \cos \theta) \) for \( a > 0 \)
   b) Inside one leaf of the four-leaved rose \( r = 3\cos(2\theta) \)
   c) Shared by the cardioids \( r = 2(1 + \cos \theta) \) and \( r = 2(1 - \cos \theta) \)
   d) Shared by the circle \( r = 2 \) and the cardioid \( r = 2(1 - \cos \theta) \)
   e) Within the inner loop of \( r = 1 - 2\sin \theta \)
   f) Inside both \( r = \sin \theta \) and \( r = 1 - \sin \theta \)

16. Determine whether the following sequence is convergent. If it converges, then find its limit.
   a) \( a_n = n\sin \frac{1}{n} \)
   b) \( a_n = \sqrt{n^2 + n} - n \)
   b) \( a_n = \left(1 + \frac{1}{n}\right)^{2n} \)
   c) \( a_n = \left(1 + \frac{5}{n}\right)^n \)

17. Determine whether the following series is convergent, if it’s convergent, then find its limit.
   a) \( \sum_{n=0}^{\infty} (3e + \pi) \)
   b) \( \sum_{n=0}^{\infty} \frac{5^{2n} - 2^{3n+1}}{5^{n+1}} \)
   c) \( \sum_{n=0}^{\infty} \left(\sqrt{n+1} - \sqrt{n}\right) \)
   d) \( \sum_{n=0}^{\infty} \left(\frac{2}{n+3} - \frac{2}{n+4}\right) \)

18. Determine whether the following series converges or diverges.
   a) \( \sum_{n=0}^{\infty} (-1)^n \frac{\ln(n+5)}{\ln(n+2)} \)
   b) \( \sum_{n=0}^{\infty} \frac{5}{n+1} \)
   c) \( \sum_{n=0}^{\infty} \frac{n^2}{n^2 + 7} \)
   d) \( \sum_{n=0}^{\infty} \frac{5}{\sqrt{2n^2 + 3}} \)
   e) \( \sum_{n=0}^{\infty} \frac{n^2 - n + 2}{2^n} (1/2) \)
   f) \( \sum_{n=0}^{\infty} \frac{(n!)^2}{n^{2n+1}} (1/e^2) \)
   g) \( \sum_{n=0}^{\infty} \left(\frac{1}{2^n} + (-1)^n \frac{1}{n}\right) \)
   h) \( \sum_{n=0}^{\infty} \frac{\sin(2n-3)}{\left(n^2 - 5n + 6\right)^3} \)
   i) \( \sum_{n=1}^{\infty} \frac{1}{n(n\ln n)^3} \)
   j) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)2^{n+3}} \)
   k) \( \sum_{n=0}^{\infty} \frac{n!}{(5n)!} \)
   l) \( \sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)2^{n+3}} \)
   m) \( \sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n} \)
19. Find the radius of convergence of the following power series.
   a) \( \sum_{n=0}^{\infty} \left( \frac{x}{2} \right)^n \)
   b) \( \sum_{n=0}^{\infty} \frac{x^n}{n!} \)
   c) \( \sum_{n=0}^{\infty} \frac{(3x)^n}{n!} \)
   d) \( \sum_{n=0}^{\infty} \frac{(x-5)^n}{n \ln n} \)
   e) \( \sum_{n=0}^{\infty} \frac{(x-3)^n}{2^n} \)
   f) \( \sum_{n=0}^{\infty} \frac{(x-2)^n}{n!} \)

20. Find a power-series representation of the function and the radius of convergence of this power series.
   a) \( f(x) = \frac{3}{x+2} \)
   b) \( f(x) = 2 \cos x + 5 \sin x \)
   c) \( f(x) = \frac{5 + 2x}{x^2 - 7x + 12} \)
   d) \( f(x) = (x^2 - 1) \sin x \)
   e) \( f(x) = \ln \left( \frac{5 + x}{1 + 3x} \right) \)
   f) \( f(x) = \ln(1 + 5x^3) \)

21. Find the first four non-zero terms on the power series representation of the following functions.
   a) \( f(x) = e^{2x} \ln(1 + x^3) \)
   b) \( f(x) = \ln(\cos x) \)

22. Find the Taylor series for \( f(x) \) centered at the given value of \( a \).
   a) \( f(x) = 1 + x + x^2, \ a = 2 \)
   b) \( f(x) = e^{3x}; \ a = -1 \)
   c) \( f(x) = \cos x; \ a = \frac{3\pi}{4} \)
   d) \( f(x) = \frac{1}{x^3}; \ a = 1 \)

23. Evaluate the following integral as an infinite series.
   a) \( \int x \cos(x^3) \, dx \)
   b) \( \int \frac{e^x - 1}{x} \, dx \)

24. Find the Maclaurin series for \( f \) and its radius of convergence.
   a) \( f(x) = \frac{x^3}{1 + 3x} \)
   b) \( f(x) = xe^{5x+3} \)

25. Find a representation of the following functions at given center \( a \) and find its radius of convergence.
   a) \( f(x) = \frac{5}{4x^3 + 3}; \ a = 2 \)
   b) \( f(x) = 3 \sin x; \ a = \frac{\pi}{3} \)

26. Using the Binomial Theorem to find the first five terms of the following functions:
   a) \( f(x) = \frac{1}{\sqrt[3]{2x-3}} \)
   b) \( f(x) = \frac{x^3}{\sqrt[4]{4-7x}} \)