1. Integrate the following integrals:

   a) \[ \int \sec^4 x \, dx \]
   b) \[ \int \cot^3 x \, dx \]
   c) \[ \int_0^{\pi/4} \frac{\sin^3 x}{\cos^4 x} \, dx \]
   d) \[ \int \frac{\sec x + \tan x}{\sec^2 x} \, dx \]
   e) \[ \int \frac{dx}{\sqrt{-16x^2 - 32x - 15}} \]
   f) \[ \int \frac{dx}{(x^2 + 4x + 5)^{3/2}} \]
   g) \[ \int_0^{1/2} \frac{dx}{\sqrt{3 - 4x - 4x^2}} \]
   h) \[ \int_0^{2} \frac{x^2 + 1}{(x^2 - 4x - 3)^{3/2}} \, dx \]
   i) \[ \int \frac{x \, dx}{(x - 2)^2} \]
   j) \[ \int \frac{x - 2}{x^2(x - 1)^2} \, dx \]
   k) \[ \int_0^{\pi/6} \frac{\cos x \, dx}{\sin x - 3 \sin x} \]
   l) \[ \int \frac{16 \, dx}{x(x^2 + 4)^2} \]
   m) \[ \int \frac{dx}{\sqrt{x - 1 + (x - 1)^3/2}} \]
   n) \[ \int \frac{dx}{\sqrt{x - 4/\sqrt{x}}} \]
   o) \[ \int \frac{\sqrt[3]{x} + 1}{\frac{1}{3}x - 1} \, dx \]
   p) \[ \int \frac{dx}{x^{2/3} - x^{3/4}} \]
   q) \[ \int x^2 \sqrt{x + 1} \, dx \]
   r) \[ \int e^{5x} \cos(7x) \, dx \]
   s) \[ \int (7x^2 - 5x + 1) \ln(5x) \, dx \]
   t) \[ \int (7x^2 + 1) \sin(4x) \, dx \]

2. Evaluate the following improper integrals

   a) \[ \int_1^\infty \frac{dx}{\sqrt{3x - 1}} \]
   b) \[ \int_{-\infty}^{0} \frac{dx}{(3x - 2)} \]
   c) \[ \int_{-2}^{2} \frac{dx}{x^2 - 4} \]
   d) \[ \int_{-\infty}^{\infty} \frac{dx}{x^2 + 1} \]
   e) \[ \int_0^{\pi/3} \tan x \, dx \]

3. Determine whether each integral is convergent or divergent.

   a) \[ \int_{-1}^{1} e^{-x^2} \, dx \]
   b) \[ \int_{1}^{\infty} \frac{dx}{x + e^{2x}} \]
   c) \[ \int_{0}^{\pi/2} \frac{dx}{x \sin x} \]

4. Find the area of the region bounded by the given curves.

   a) \[ y = \frac{1}{x^2} \text{, } y = 0 \text{, right of } x = 2 \]
   b) \[ y = \frac{1}{(x - 1)^{1/3}} \text{, } y = 0 \text{ between } x = 0 \text{ and } x = 2 \]

5. Prove the following:

   a) Show that \[ \int_0^1 \frac{dx}{x^p} \] converges if and only if \( p < 1 \).
   b) Show that \[ \int_1^\infty \frac{dx}{x^p} \] converges if and only if \( p > 1 \)

6. Find the values of \( p \) for which each integral converges:

   a) \[ \int_1^2 \frac{dx}{(x(ln x))^p} \]
   b) \[ \int_2^\infty \frac{dx}{x(ln x)^p} \]

7. Sketch the isocline for the following:

   a) \[ \frac{dy}{dx} = 3x - 5y \]
   b) \[ \frac{dy}{dx} = y^2(y - 3)(y + 1) \]

8. Find the solution of the differential equation that satisfies the given initial condition.

   a) \[ \frac{dy}{dx} = (1 + y^2) \tan x; \quad y(0) = \sqrt{3} \]
   b) \[ \frac{dy}{dx} = 8x^3 e^{-2y}; \quad y(1) = 0 \]
   c) \[ \sqrt{y} \, dx + (1 + x) \, dy = 0; \quad y(0) = 1 \]
   d) \[ x^2 \, dx + 2y \, dy = 0; \quad y(1) = 0 \]
   e) \[ y^{-1} \, dy + ye^{\cos x} \sin x \, dx = 0 \]
   f) \[ (x + xy^2) \, dx + e^{x^2} \, y \, dy = 0 \]
   g) \[ x \, \frac{dy}{dx} + 2y = 5x^3 \]
   h) \[ \left(x^2 + 1\right) \frac{dy}{dx} = x^2 + 2x - 1 - 4xy \]
   i) \[ \cos x \, \frac{dy}{dx} + y \sin x = 2x \cos^2 x; \quad y \left(\frac{\pi}{4}\right) = \frac{-15\sqrt{2}\pi^2}{32} \]
   j) \[ \frac{dy}{dx} = x^2 e^{-x} - 4y \]
9. Solve the following application problems:
   a) A bank account earns 5% annual interest compounded continuously. You wish to make payments out of the account at a rate of $12,000 per year (in a continuous cash flow) for 20 years.
      i) Write a differential equation describing the balance $B = f(t)$, where $t$ is in years.
      ii) Find the solution $B = f(t)$ to the differential equation given an initial balance of $B_0$ in the account.
      iii) What should the initial balance be such that account has zero balance after precisely 20 years?
   b) A rectangular swimming pool has dimensions, in meters, of 20 by 10 by 10; hence it has volume 2000 cubic meters, or $2 \times 10^6$ liters. The pool initially contains pure fresh water. At time $t = 0$ minutes, water containing 10 grams/liter of salt is poured into the pool at a rate of 60 liters/minute. The salt water is instantly and totally mixed with the fresh water, and the excess mixture is drained out of the bottom of the pool at the same rate (60 liters/minute). Let $S(t) =$ the mass of salt in the pool at time $t$.
      i) Write a differential equation for the amount of salt in the pool.
      ii) Solve the differential equation to find $S(t)$
      iii) What happens to $S(t)$ as $t \to \infty$?
   c) Suppose a brine containing 0.2kg of salt per liter runs into a tank initially filled with 500 L of water containing 5 kg of salt. The brine enters the tank at a rate of 5 L/min. The mixture, kept uniform by stirring, is flowing out at the rate of 5 L/Min.
      i) Find the concentration, in kilogram per liter, of salt in the tank after 10 min.
      ii) After 10 min, a leak develops in a tank and an additional liter per minutes of mixture flows out of the tank. What will the concentration, in kilograms per liter, of salt in the tank 20 minutes after the leak develops?
   d) $1000$ is put into a bank account and earns interest continuously at a rate of I per year, and in addition, continuous payments are made out of the account at a rate of $100$ a year. Sketch the amount of money in the account as a function of time if the interest rate is (i) 5% (ii) 10% (iii) 15%
   In each case, you should first find an expression for the amount of money in the account at time $t$ (in years)
   e) A bank account earns 5% annual interest compounded continuously. You wish to make payments out of the account at a rate of $12,000$ per year (in a continuous cash flow) for 20 years.
      i) Write a differential equation describing the balance $B = f(t)$, where $t$ is in years.
      ii) Find the solution $B = f(t)$ to the differential equation given an initial balance of $B_0$ in the account.
      iii) What should the initial balance be such that account has zero balance after precisely 20 years?

10. Find $\frac{dy}{dx}$ and $\frac{d^2 y}{dx^2}$ of the following parametric equations
   a) $x = 5t^3 - 2t + 1; \quad y = t^2 - t + 3$
   b) $x = 5 + t \sin t; \quad y = 2 - t^2 \cos t$

11. Find the points on the curve where the tangent is horizontal or vertical.
   a) $x = t - 2 \sin t; \quad y = 2 - \cos t$
   b) $x = 4 + 2 \cosh t; \quad y = 1 - 4 \sinh t$

12. Find an equation of the tangent to the curve at the given point by two methods: (a) without eliminating the parameter and (b) by first eliminating the parameter.
   a) $y = e^t; \quad y = (t - 1)^2; \quad (1,1)$
   b) $x = \tan t; \quad y = \sec t; \quad \left(1, \sqrt{2}\right)$

13. Sketch the curves
   a) $x = \theta - \sin \theta; \quad y = 1 - \cos \theta$
   b) $x = \cos t + t \sin t; \quad y = \sin t - t \cos t$

14. Graph the curve of the following polar equations:
   a) $r = 1 - \cos \theta$
   b) $r = 5 + 7 \cos \theta$
   c) $r = 3 \cos(3\theta)$
   d) $r = 5 \sin(4\theta)$
   e) $r = 2 - 3 \cos(\theta)$
   f) $r = 2 + 3 \sin \theta$

15. Convert the following polar equations to Cartesian equations
   a) $r = 4 \csc \theta$
   b) $r^2 \sin(2\theta) = 2$
   c) $r = (\csc \theta)e^{r \cos \theta}$
   d) $r = 2 \cos \theta + 2 \sin \theta$
   e) $r^2 = -4r \cos \theta$
   f) $r = \frac{5}{\sin \theta - 2 \cos \theta}$
16. Find the intersections of the following curves
   a) \( r = 1 + \cos \theta \) and \( r = 1 - \cos \theta \)  
   b) \( r = 2 \sin \theta \) and \( r = 2 \sin(2\theta) \)  
   c) \( r = \cos \theta \) and \( r = 1 - \cos \theta \)

17. Find the horizontal and vertical tangents to the graph of the following:
   a) \( r = -1 + \sin \theta \) for \( 0 \leq \theta \leq 2\pi \)  
   b) \( r = 3 - 4 \cos \theta \) for \( 0 \leq \theta \leq 2\pi \)

18. Find the area of the following regions:
   a) Inside the cardioid \( r = a(1 + \cos \theta) \) for \( a > 0 \) 
   b) Inside one leaf of the four-leaved rose \( r = 3 \cos(2\theta) \)  
   c) Shared by the cardioids \( r = 2(1 + \cos \theta) \) and \( r = 2(1 - \cos \theta) \) 
   d) Shared by the circle \( r = 2 \) and the cardioid \( r = 2(1 - \cos \theta) \) 
   e) Within the inner loop of \( r = 1 - 2 \sin \theta \)  
   f) Inside both \( r = \sin \theta \) and \( r = 1 - \sin \theta \)