Worksheet #2

Math 160

Name:

- 1. Sketch the graph of the following functions:
 - a) $f(x) = \left(\frac{3}{7}\right)^{x-5}$ b) $f(x) = -5e^{2-3x}$ c) $f(x) = 2\left(\frac{3}{5}\right)^{-3x}$
- 2. Solve for x:
 - a) $3^{5x-3} = \frac{1}{27}$ b) $5^{2x-7} = 3^{2-3x}$ c) $2^{3-5x} = e^{x+1}$
 - d) $3 \cdot 2^{7x+3} = 7$ e) $\log_2 \frac{3-5x}{7-2x} = -3$ f) $\log_3 (\log_2 (3x-8)) = -2$
 - g) $\log_2(7-2x) \log_2(5-3x) = -2$ h) $\ln(x+1) = 2 + \ln(x-1)$
 - i) $\log(x-6) + \log(x+3) = 1$ k) $\log(x+1) = 2\log(x-1)$
- 3. Express as a single logarithm with a coefficient of 1:
 - a) $\ln(x^2 9) + 2\ln\frac{1}{x+3} + 4\ln x$ b) $4\log 3 6\log(x^2 + 1) + \frac{1}{2}\left[\log(x+1) 2\log 3\right] + 1$
- 4. Write the quantity using sums and differences of simpler logarithmic expressions. Express the answer so that logarithms of products, quotients, and powers do not appear.

a)
$$\ln\left(\sqrt{\frac{(x+1)(x+2)}{(x-1)(x-2)}}\right)$$
 b) $\ln\left(\frac{x\sqrt[3]{4x+1}}{\sqrt{2x-1}}\right)$

- 5. Solve the following application problems:
 - a) Suppose that \$3500 is invested at 7.5% interest compounded monthly. How many years will it take for the money to double?
 - b) What principal should you deposit at 5.5% per annum compounded semiannually so as to have \$6000 after 10 years?
 - c) You have two savings accounts, each with an initial principal of \$1,000. The nominal rate on both accounts is 5.5% per annum. In the first account, interest is compounded semiannually. In the second account, interest is compounded continuously. How much more is in the second account after 12 years?
 - d) Hospitals utilize the radioactive substance iodine-131 in the diagnosis of conditions of the thyroid gland. The half-life of iodine-131 is eight days.
 If a hospital acquires 2 g of iodine-131, how much of this sample will remain after 20 days? How long will it be until only 0.01 g remains?
 - e) A child's grandparents are considering buying a \$40,000 face value zero-coupon bond at birth so that she will have enough money for her college education 17 years later. If they want a rate of return of 8% compounded annually, what should they pay for the bond?
 - f) A piece of charcoal is found to contain 30% of the carbon 14 it originally had. When did the tree from which the charcoal came die?
 - g) A fossilized leaf contains 70% of its normal amount of carbon 14. How old is the fossil?
 - h) A pizza baked at 450° F is removed from the oven at 5:00pm in to a room that is a constant 70° F.
 After 5 minutes, the pizza is at 300° F. At what time can you begin eating the pizza if you want its temperature to be 135° F? Determine the time that needs to elapse before the pizza is 160° F?
- 6. Fill out the following table by using the unit circle.

θ	3π	7π	23π	5π	13π	7π	23π	11π	25π
	4	6	3	3	4	2		6	3
$\sin \theta$									
$\cos \theta$									
$\sin \theta$									
$tan \theta$									
$\csc \theta$									
$\sec\theta$									

- 7. Use the unit circle to find all values of θ between 0 and 2π for which
 - a) $\sin \theta = \frac{1}{2}$ b) $\cos \theta = -\frac{\sqrt{3}}{2}$ c) $\sin(-\theta) = \frac{\sqrt{2}}{2}$ d) $\cot \theta = -1$ e) $\sec \theta = -2$ f) $\csc \theta = -\sqrt{2}$
- 8. a) The minute hand of a clock is 3.5 cm long. How far does the tip of the minute hand travel in 25 minutes?
- b) A mixing blade on a food processor extends out 3 inches from its center. If the blade is turning at 600 rpm, what is the linear velocity of the tip of the blade in feet per minute?
- c) A 5-inch fixed disk in a computer rotates at 3,600 rpm. Find the linear velocity of a point 2 inches from the center of the disk. Then find the linear velocity of a point 1 inch from the center.
- d) A woman rides a bicycle for 1 hour and travels 16 km (about 10 mi). Find the angular velocity of the wheel if the radius is 30 cm.

f)

- e) A propeller with radius 1.50 ft is rotating at 900 rpm. Find the linear velocity of the tip of the propeller.
- 9. Prove the following identities:

a)
$$\sec^2 x - \tan^2 x = 1$$

b) $\frac{1}{1 + \cos x} + \frac{1}{1 - \cos x} = 2\csc^2 x$

c)
$$\frac{\sin x + 1}{\cos x + \cot x} = \tan x$$
 d)

e)
$$\tan x \sec x \cos x = 1$$

g) Show that
$$\cos(\tan^{-1} v) = \frac{1}{\sqrt{1 + v^2}}$$
 h)

$$\frac{\tan x}{\sec x} = \sin x$$

$$\frac{\left(2\cos^2 x - 1\right)^2}{\cos^4 x - \sin^4 x} = \frac{1 + \cos x}{\sin x}$$

$$\frac{1 + \cos x + \sin x}{1 + \cos x - \sin x} = \sec x + \tan x$$

10. Sketch the graph of the following functions over one period.

a)
$$f(x) = \frac{1}{2}\cos(5x-3)+1$$

b) $f(x) = -\frac{1}{3}\csc(7x-\pi)+2$
c) $f(x) = 3\tan(7-2x)-1$
d) $f(x) = -3\sin(5x-\pi)+1$

11. Find the exact value of the following without using a calculator.

a)
$$\cos\left(\tan^{-1}\frac{3}{4}\right)$$
 b) $\sec\left(\cos^{-1}\frac{1}{2}\right)$ c) $\cos\left(\sin^{-1}\frac{7}{9}\right)$
d) $\cos\left(\sin^{-1}\left(-\frac{1}{3}\right) - \tan^{-1}(3)\right)$ e) $\sin\left(2\cos^{-1}\left(-\frac{\sqrt{2}}{5}\right)\right)$ f) $\cos\left(\frac{1}{2}\tan^{-1}\left(-\sqrt{5}\right)\right)$
g) $\sin\left(\tan^{-1}\left(-\sqrt{5}\right) + \cos^{-1}\left(\frac{1}{3}\right)\right)$ h) $\tan\left(\cos^{-1}\left(-\frac{1}{4}\right) - \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)\right)$
Solve the following equations:
a) $-2\sin(5x - \pi) = 1$ b) $\sin(2\theta) + \sin(4\theta) = 0$
c) $\sqrt{3}\sin\theta + \cos\theta = 1$ d) $\sin^{2}\theta = 6(\cos\theta + 1)$

c)
$$\sqrt{3}\sin\theta + \cos\theta = 1$$

d) $\sin^2\theta = 6(\cos\theta + 1)$
e) $2\sin^2\theta - 3\sin\theta + 1 = 0$
f) $\sin(2\theta) = \sqrt{2}\cos\theta$

13. Solve the following application problems.

12.

b) A security camera in a neighborhood bank is mounted on a wall 9 ft above the floor. What angle of depression should be used if the camera is to be directed to a spot 6 ft above the floor and 12 ft from the wall?c) From a point on ground level, you measure the angle of elevation to the top of a mountain to be 38 degree. Then you walk 200 m farther away from the mountain and find that the angle of elevation is now 20 degree. Find the height of the mountain. Round the answer to the nearest meter.