Worksheet #1

1. Given the graph of a function \( f(x) \). Evaluate the following:
   a) \( \lim_{x \to 5} f(x) = \)
   b) \( \lim_{x \to 2} f(x) = \)
   c) \( \lim_{x \to -1} f(x) = \)
   d) \( \lim_{x \to -3} f(x) = \)
   e) \( \lim_{x \to -4} f(x) = \)
   f) \( \lim_{x \to \infty} f(x) = \)

2. Evaluate the following limits:
   a) \( \lim_{x \to -2/7} \frac{21x^2 - 29x - 10}{7x + 2} \)
   b) \( \lim_{x \to 3} \frac{2}{(x - 3)^2} \)
   c) \( \lim_{h \to 0} \frac{(x + h + 1)^2 - (x + 1)^2}{h} \)
   d) \( \lim_{x \to 5} \frac{-7}{(x - 5)^2} \)
   e) \( \lim_{x \to 3} \frac{2x^2 - 11x + 15}{3x^2 - 5x - 12} \)
   f) \( \lim_{x \to -1} \frac{\sqrt{x^2 + 8} - 3}{x + 1} \)
   g) \( \lim_{x \to 2} f(x) \) where \( f(x) = \begin{cases} 5x^2 - 6x - 6; & x \geq 2 \\ e^{\sin \left( \frac{x}{2} \right)} \log \left( 50x^2 - 25 - 50 \right); & x < 2 \end{cases} \)
   h) \( \lim_{x \to 0} \left( \frac{3}{2x} - \frac{3}{2|x|} \right) \)
   i) \( \lim_{x \to 1} \frac{\sqrt{2}x(x - 1)}{|x - 1|} \)
   j) \( \lim_{x \to \pi} \tan (3x - \pi) \)
   k) \( \lim_{x \to 3} \log (3x^2 - 5x - 2) \)
   l) \( \lim_{x \to 0} x^6 \cos \left( \frac{1}{5x} \right) \)
   m) \( \lim_{x \to \infty} \frac{\cos^5 (2x - 1)}{x^2} \)
   n) \( \lim_{x \to \infty} \tan^{-1} \left( \frac{\sqrt{3}e^{3x} + 2e^{-3x}}{e^{3x} - 4e^{-3x}} \right) \)

3. Sketch a possible graph of the following function:

\[
\lim_{x \to -3} f(x) = \infty; \quad f(2) = 1; \quad \lim_{x \to -2} f(x) = 5; \quad \lim_{x \to 2} f(x) = -1; \quad \lim_{x \to \infty} f(x) = -2; \quad \lim_{x \to -\infty} f(x) = -\infty
\]

4. Determine whether the function is continuous at \( x = a \)
   a) \( f(x) = \begin{cases} x^3 \sin \left( \frac{\pi x}{4} \right); & x < 2 \\ 3x^2 + 4x - 4 \div x + 2; & x \geq 2 \end{cases} \) \( a = 2 \)
   b) \( f(x) = \begin{cases} \sqrt{2x^2 - 2x + 10}; & x \neq 0 \\ \frac{3}{7x - 6}; & x = 0 \end{cases} \)

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5. Determine the value(s) for a such that the following function is continuous every where.

\[
 f(x) = \begin{cases} 
 (3x^2 - ax + 4) \sin \left( \frac{\pi x}{6} \right) & : x \neq 2 \\
 a - 2x & : x = 2
\end{cases}
\]

a) \( f(x) = \left\{ \begin{array}{l} 
(3x^2 - ax + 4) \sin \left( \frac{\pi x}{6} \right) \\
 a - 2x
\end{array} \right. \); \( x \neq 2 \)

b) \( f(x) = \left\{ \begin{array}{l} 
(2x^2 + ax - r) & : x \geq x_o \\
r + 4x & : x < x_o
\end{array} \right. \)

6. Prove by \( \varepsilon - \delta \) definition for \( \lim_{x \to a} f(x) = L \).

a) \( \lim_{x \to 3} (3x - 9) = 6 \)  

b) \( \lim_{x \to 3} \frac{x^2 - 7x + 12}{x^2 - 9} = -\frac{1}{6} \)

c) \( \lim_{x \to 3} (2x^2 - 5x + 1) = 4 \)

7. Find the slope of tangent line to the following curve at given points:

a) \( f(x) = 3x^2 - x + 4; \ at \ x = 2 \)

b) \( f(x) = \frac{3x - 5}{7 - 2x}; \ at \ x = -2 \)

c) \( f(x) = \frac{1}{\sqrt{x + 1}}; \ at \ x = 0 \)

d) \( f(x) = (x - 1)^3; \ at \ x = 2 \)

8. Use the IVT to show the following have at least one solution over the indicated intervals.

a) \( f(x) = 2x^3 - \pi x + 1; \ [1, 2] \)

b) \( f(x) = 2x^5 - 7x + 3; \ [1, 2] \)

c) \( 3x^3 = 2 \sin (3x) + 1 \) over \( \mathbb{R} \).

c) \( 5 \cos (2x) = x^7 - 3 \) over \( \mathbb{R} \).

9. Find equation of tangent line to following curves at given points:

a) \( f(x) = \frac{7x^3 - 3}{5 - 2x}; \ at \ x = 4 \)

b) \( f(x) = 7x^2 - 5x + 1; \ at \ x = 4 \)

c) \( f(x) = \sin (3x); \ at \ x = \frac{\pi}{6} \)

10. Find the point(s) on the curve where the slope of tangent line is -3.

a) \( f(x) = 7x^2 - 5x + 1 \)

b) \( f(x) = -\sqrt[3]{x} = 1 \)

11. Find the derivative of the following functions:

a) \( f(x) = \sqrt{\frac{\sin (3x + 2)}{x^4 - 5x + 2}} \)

b) \( f(x) = \sin^3 \left( 4x^5 + 2x^3 - 2 \right) \sqrt[4]{2x^4 - 7x^2 + 2} \)

c) \( f(x) = \sin \left( \sin \left( 7x^3 + 2x - 4 \right) \right) \)

12. Find the equation of a tangent line to the following functions at given points.

a) \( f(x) = 1 \) \( \sqrt{x^2 + 1} \) at \( \left( 1, \frac{1}{\sqrt{2}} \right) \)

b) \( f(x) = x \cos x + \sin x \) at \( x = \frac{\pi}{3} \)

c) \( f(x) = \sqrt{\frac{3x^2 - 5x + 1}{x^2 + 1}} \) at \( x = 0 \)

d) \( f(x) = e^x \cos (7x) - \sin 2x + 1 \) at \( x = 0 \)

13. Prove the following statements:

a) \( \frac{d}{dx} (\sin x) = \cos x \)

b) \( \frac{d}{dx} (\cos x) = -\sin x \)

c) \( \frac{d}{dx} (\tan^{-1} x) = \frac{1}{1 + x^2} \)

14. The position function of an object is given by \( s(t) = 4 \sin (0.8t) \). Find the velocity and acceleration as functions of time. What is the range of the position, velocity and acceleration functions?
15. Find \( \frac{dy}{dx} \) by implicit differentiation.
   
   a) \( (3x^2 - x + 2y^3 - 1)^2 + 3y - 2x + 1 = 0 \)
   
   b) \( \cos(5y - 7x + 3) - x^2y + y^2 = 5 \)
   
   c) \( (7x - y + 1)\tan(5x + xy^2 - 3) = 1 \)
   
   d) \( \sec(x + y) = \tan x - \tan y \)
   
   e) \( \cos \left( \sin \left( 5x^2 - 2y - 1 \right) - x^2 + y - 2 \right) - x^2y = x + y^3 \)
   
   f) \( \frac{3x \sin (2y - 1)}{x^2 - y \cos y} - 2xy^2 = x - y - 1 \)

16. Find an equation of the tangent line to the following equations at given points.
   
   a) \( x^2 + y^2 - 6x - 8y = 0 \) at \( (6, 0) \)
   
   b) \( x^2 + y^2 = (2x^2 + 2y^2 - x)^2 \) at \( (0, \frac{1}{2}) \)
   
   c) \( y^2 = 5x^4 - x^2 \) at \( (1, 2) \)

17. a) Find equations of the lines tangent to \( x^2 + y^2 - 6x - 4y - 12 = 0 \) and containing the point \( (-4, 3) \)
   
   b) At what point(s) does \( 3x^2 + y^2 + 4x - 6y + 1 = 0 \) have a horizontal tangent? A vertical tangent?

18. Show that the line tangent to \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) at \( (x_0, y_0) \) is \( \frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1 \)

19. Find the third derivative of the following functions:
   
   a) \( f(x) = \frac{1}{2x - 5} \)
   
   b) \( f(x) = \sin(\sin x) \)

20. a) A point is moving along the curve \( y = \sqrt{x} \) in such a way that its x coordinate is increasing at the rate of 3 units per minute. At what rate is y changing (a) when \( x = 1 \)? (b) when \( x = 4 \)?

   b) A tank has a conical bottom of radius 4 m and height 3 m. It is surmounted by a cylinder of radius 4 m and height 10 m. Water is pumped in at the rate of 8 m\(^3\)/min. How fast is the water rising when the depth (from the apex of the cone) is 2 m? When the depth is 5 m?

   c) Ship A is steaming north at 10 mph. Ship B, which is 8 miles west of ship A, is steaming south at 12 mph. At what rate is the distance between them changing? At what rate will it be changing one hour from now?

   d) A lighthouse is 2 miles off a straight shore. Its light makes three revolutions per minute. How fast does t from the light beam move along a sea wall at a point 2 miles down the coast?

   e) A person 1.8 m tall is walking away from a lamppost 6 m high at the rate of 1.3 m/sec. At what rate is the end of the person’s shadow moving away from the lamppost? At what rate is the end of the shadow moving away from the person?