Assignment #2 ***Math 180*** ***Name:***

1. Evaluate the following:

a) $\lim_{x \to \frac{\pi}{2}^-} \sec(3x)$

b) $\lim_{x \to \infty} \frac{7x^3 - 5x + 4}{2x^3 + x^2 - 3}$

c) $\lim_{x \to \infty} \frac{\cos^2(3x - \pi)}{\sqrt{x^2 + 1}}$

d) $\lim_{x \to \infty} \frac{\sqrt{4x^6 - 5x^3 - 3}}{2x^2 - 3x + 4}$

e) $\lim_{x \to \infty} \left[\ln(3x - 1) - \ln(2x + 1)\right]$

f) $\lim_{x \to \infty} \frac{1}{\sqrt{x^2 + 1} - \sqrt{x^2 + 4x}}$

g) $\lim_{x\to\infty} \tan^{-1}(7x-1)$

h) $\lim_{h\to 0} \dfrac{\dfrac{x+h+1}{x+h} - \dfrac{x+1}{x}}{h}$

2. a) For $f(x)=x^2$, evaluate $\lim_{h\to 0} \dfrac{f(1+h)-f(1)}{h}$ *and* $\lim_{x\to 1} \dfrac{f(x)-f(1)}{x-1}$

b) Suppose $f(x)=\begin{cases} 2x^2+rx-r & if \;\; x \ge x_0 \\ r+4x & if \;\; x < x_0 \end{cases}$; find r so that the $\lim_{x\to x_0} f(x)$ exists.

3. Using $\varepsilon - \delta$ definition of limit to prove the following:

a) $\lim_{x \to -4} (2x + 11) = 3$

b) $\lim_{x \to 2} (3x^2 - 5x + 1) = 3$

c) $\lim_{x \to \frac{1}{2}} \frac{2x^2 + 7x - 4}{2x - 1} = \frac{9}{2}$

4. Determine if f(x) is continuous, removable discontinuous or jumped discontinuous at x=a

a) $f(x) = \begin{cases} \sqrt{3x^2 + 4} - 2; \ if \quad x \geq 2 \\ 2\cos\left(\frac{\pi x}{6}\right) + 2x - 3; \ if \ x < 2 \end{cases}; \ a = 2$

b) $$f(x)=\begin{cases}\dfrac{2x^2+5x-3}{2x^2-9x+4}; \ if \ \ x\neq\dfrac{1}{2} \\ \quad\dfrac{4}{3}; \qquad if \ \ x=\dfrac{1}{2}\end{cases}; \ a=\dfrac{1}{2}$$

c) $$f(x)=\begin{cases}\tan^{-1}\left(\sqrt{x}\right)+2x; \ \ if \ \ x<3 \\ 2x^2-x-3; \qquad if \ \ x\geq 3\end{cases}; a=3$$

5. Under what conditions for the following functions to be continuous for all real numbers.

a) $$f(x)=\begin{cases}\sqrt{7x+k}; & if \;\; x>2 \\ x^2+kx+4; & if \;\; x\leq 2\end{cases}$$

b) $$f(x)=\begin{cases}x^2; & if \quad x<-2 \\ ax^2+bx+1; & if \quad -2\leq x\leq 2 \\ x^2+2; & if \quad x>2\end{cases}$$

6. Using IVT to prove the following equations have at least one real solution.

a) $e^{2x} = 2\sin(3x) + 5$

b) $3x^3 + 1 = e^{2-3x}$