Assignment #5 ***Math 260*** ***Name:***

1. Determine whether a vector belongs to span(S)

a) $\vec{v} = \begin{bmatrix} -2 \\ 3 \\ 3 \end{bmatrix} \in Span(S) \text{ where } S = \left\{ \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 3 \end{bmatrix} \right\}$

b) $f(x) = 4x^2 + 6x + 4 \in Span(S) \text{ where } S = \left\{ 2x^2 - 4, x^2 - x - 4 \right\}$

c) $\vec{v} = \begin{bmatrix} 9 & -3 \\ -5 & 5 \end{bmatrix} \in Span(S) \text{ where } S = \left\{ \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}; \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}; \begin{bmatrix} 3 & 1 \\ 0 & 0 \end{bmatrix} \right\}$

d) $\overrightarrow{v} = 7x^3 - 2x^2 + 2 \in Span(S) \text{ where } S = \{x^2 + 1, x^3 - 2, x + 3\}$

e) $A = \begin{bmatrix} 2 & 0 \\ 2 & -1 \end{bmatrix} \in Span(S) \text{ where } S = \left\{ \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}; \begin{bmatrix} -1 & 0 \\ -2 & 3 \end{bmatrix}; \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix} \right\}$

2. Determine whether the following set of vectors are linearly independent or linearly dependent.

a) $S = \left\{ \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ -4 \end{bmatrix} \right\}$

b) $S=\{7x-2, 3x^2+x-3, 2x^2-1\}$

c) $S=\left\{\begin{bmatrix}1\\2\\3\end{bmatrix},\begin{bmatrix}4\\5\\6\end{bmatrix},\begin{bmatrix}7\\8\\9\end{bmatrix}\right\}$

d) $S=\{7x^2-5, 4x^2+x-1, 2x-3\}$

e) $S = \left\{ \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}; \begin{bmatrix} -4 & -1 \\ 0 & 5 \end{bmatrix}; \begin{bmatrix} -8 & -3 \\ -6 & 17 \end{bmatrix} \right\}$

3. Let $S = \left\{ \overrightarrow{v_1}, \overrightarrow{v_2}, \ldots, \overrightarrow{v_n} \right\}$ be a set of linearly independent vectors. Prove that $\left\{ \overrightarrow{v_1}, \overrightarrow{v_1} + \overrightarrow{v_2}, \overrightarrow{v_1} + \overrightarrow{v_2} + \overrightarrow{v_3}, \ldots, \overrightarrow{v_1} + \overrightarrow{v_2} + \ldots + \overrightarrow{v_n} \right\}$ is also a linearly independent set of vectors.