

1. Find a linearly independent minimal set that spans $\text{span}(S)$.

a)
$$S = \left\{ \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}; \begin{bmatrix} 1 \\ 4 \\ -9 \end{bmatrix}; \begin{bmatrix} 3 \\ 3 \\ -9 \end{bmatrix} \right\}$$

b)
$$S = \{3x^2 - 2x + 1, 4x^2 - 3, x^2 - 6x + 9, 4x^2 - 8x + 10\}$$

c)
$$S = \left\{ \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix}; \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} -7 & 4 \\ -3 & -6 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 2 & 4 \end{bmatrix}; \begin{bmatrix} -6 & 4 \\ -2 & -4 \end{bmatrix} \right\}$$

2. Find a basis and dimension of the following vector spaces:

a)
$$V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in R^3 \mid 3a - 2b + c = 0 \right\}$$

b)
$$V = \left\{ A \in M_2(R) \mid A^T = A \right\}$$

c)
$$V = \left\{ f(x) \in P_2 \mid \int_0^1 f(x) dx = 0 \right\}$$

d) $V = \{f(x) \in P_3 \mid f(0) + f'(1) + f''(2) = 0\}$

e) $V = \text{Null space of } A = \text{NullSpace}(A) \text{ where } A = \begin{bmatrix} 3 & -1 & 1 \\ 5 & 0 & 5 \\ 2 & 1 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

3. Find a basis of the following subspace S and then extend their basis to be a basis of vector space V.

a)
$$S = \text{span} \left\{ \begin{bmatrix} 3 \\ -1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 7 \\ -5 \\ 4 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 10 \\ -6 \\ 6 \\ -6 \end{bmatrix} \right\}$$

b)
$$S = \{y \in P_3 \mid y(0) + y'(1) = 0\}$$

c) $S = \{A \in M_2(R) \mid A^T = A\}$

d) $S = \text{span}\{1+x+2x^3, 2+x+3x^2-x^3, -1+x+x^2-2x^3, 2-x+x^2+2x^3\}$