

1. Find a basis of rowspace and colspace of the following matrices.

a) $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 7 & 8 & 9 & 10 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 2 & -1 & -2 & -1 \\ 2 & 4 & -2 & -3 & -1 \\ 5 & 10 & -5 & -3 & -1 \\ -3 & -6 & 3 & 2 & 1 \end{bmatrix}$

2. Find the rank, and nullity of the following matrices:

a)
$$A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & 4 & 4 \\ 1 & 1 & 0 \end{bmatrix}$$

b)
$$A = \begin{bmatrix} 2 & -3 \\ 0 & 0 \\ -4 & 6 \\ 22 & -33 \end{bmatrix}$$

3. Consider a system of linear equations in the form $A\vec{x} = \vec{b}$, determine whether $\vec{b}$ is in the column space of A. If it is, write the solution in the form $\vec{x} = \vec{x}_h + \vec{x}_p$

a)
$$\begin{cases} 2x - 4y + 5z = 8 \\ -7x + 14y + 4z = -28 \\ 3x - 6y + z = 12 \end{cases}$$

b)
$$\begin{cases} 3w - 2x + 16y - 2z = -7 \\ -w + 5x - 14y + 18z = 29 \\ 3w - x + 14y + 2z = 1 \end{cases}$$

4. Determine whether $\vec{b}$ is in the column space of A. If it is write $\vec{b}$ as a linear combination of the column vectors of A.

a) $A = \begin{bmatrix} 3 & -3 & -1 \\ 1 & 1 & -2 \\ 2 & 4 & 5 \end{bmatrix}; \vec{b} = \begin{bmatrix} 11 \\ -9 \\ 12 \end{bmatrix}$

b) $B = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 2 & 2 \\ -1 & 1 & 0 \\ 4 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \\ -3 \\ 22 \end{bmatrix}$