

1. Using the standard inner product in R^n to determine the angle between vectors:

a) $\vec{v} = \langle 0, -2, 1, 4, 1 \rangle$ and $\vec{u} = \langle -3, 1, -1, 0, 3 \rangle$

b) $\vec{v} = \langle 2, -2, 0, 4 \rangle$ and $\vec{u} = \langle 1, 2, -2, 0 \rangle$

2. Given $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ from $B = M_2(R)$. Prove / disprove if the following is an inner product vector space (IPVS)

a) $\langle A, B \rangle = a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + a_{22}b_{22}$

b) $\langle A, B \rangle = a_{11}b_{11}$

3. Given $f_1(x) = a_1x^2 + b_1x + c_1$ and $f_2(x) = a_2x^2 + b_2x + c_2$ from $V = P_2$

a) $\langle f_1(x), f_2(x) \rangle = a_1a_2 + b_1b_2 + c_1c_2$

b) $\langle f_1(x), f_2(x) \rangle = \int_0^1 xf_1(x)f_2(x)dx$

4. Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ from $B = M_2(R)$. Define

$$\langle A, B \rangle = 2a_{11}b_{11} + a_{12}b_{12} + a_{21}b_{21} + 2a_{22}b_{22}$$

Determine $\langle A, B \rangle$; $\|A\|$; $\|B\|$ and angle between A and B.

a) $A = \begin{bmatrix} 2 & -4 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$

b) $A = \begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -6 & 2 \\ 4 & -10 \end{bmatrix}$