Practice for Sequences: ***Name:***

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

a) $\left\{\dfrac{n}{1+\sqrt{n}}\right\}$

b) $\left\{\dfrac{(2n-1)!}{(2n+1)!}\right\}$

c) $a_n = \sqrt{n} - \sqrt{n^2 - 1}$

d) $a_n = \left(1 + \dfrac{2}{n}\right)^{1/n}$

e) $a_n = \ln(n+1) - \ln n$

f) $a_n = \dfrac{(-1)^{n+1} n^2}{n^2 + 1}$

2. Find the limit of the sequence $\left\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \sqrt{2\sqrt{2\sqrt{2\sqrt{2}}}} \ldots\right\}$

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3. Show that the sequence defined by $a_1 = 2;\ a_{n+1} = \dfrac{1}{3 - a_n}$ satisfies $0 < a_n \le 2$ and is decreasing. Deduce that the sequence is convergent and find its limit.

4. Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

a) $a_n = \dfrac{3n-7}{7n+5}$

b) $a_n = \cos^2\left(\dfrac{n\pi}{2}\right)$

c) $a_n = ne^{-n}$

d) $a_n = \dfrac{n-3}{n^2+1}$

e) $a_n = \dfrac{2^n 3^n}{n!}$

f) $a_n = 2 - \dfrac{2}{n} - \dfrac{1}{2^n}$