

1. Determine whether the following DE has a unique solution:

a) $\frac{dy}{dx} = \sin(3x + y); y(\pi) = 0$

b) $\frac{dy}{dx} = (4x - y)^{2/3}; y(0) = 0$

c) $\frac{dy}{dx} = \tan(xy); y\left(\frac{\pi}{3}\right) = 1$

2. Verify $y(x)$ is a solution, then use the initial values to determine the constants c_1 and c_2

a) $y = e^{-2x} \left(c_1 \cos(\sqrt{5}x) + c_2 \sin(\sqrt{5}x) \right)$ for $y'' + 4y' + 9y = 0$; $y(0) = 1$, $y'(0) = -2$

b) $y = c_1 e^{3x} + c_2 e^{-x} - 2x$ for $y'' - 2y' - 3y = 6x + 4$ $y(1) = 4$ and $y'(1) = -2$

3. Sketch the phase-diagram, isocline of the following autonomous DE, then determine stable, unstable and semi-stable equilibrium solutions:

a) $\frac{dy}{dx} = -3y^2 + 17y - 10$

b) $\frac{dy}{dx} = y^3 - 5y^2 - 4y + 20$

c) $\frac{dy}{dx} = -2y^3 + 14y^2 - 30y + 18$

4. Sketch the isoclines of the following DE:

a)
$$\frac{dy}{dx} = -\frac{x}{y}$$

b)
$$\frac{dy}{dx} = \frac{1}{2}y - x$$