Show that $7x = 4\cos(x) - 10$ has exactly one real solution.

\[ 7x - 4\cos(x) + 10 = 0 \]

Define $f(x) = 7x - 4\cos(x) + 10 \implies f(x)$ is continuous and differentiable over $\mathbb{R}$.

Suppose $f(x)$ has 2 real solutions, say $x = a < x = b$.

Since $x = a$ is a sol'n $\implies f(a) = 0$ \& $f(a) = f(b)$

$x = b$ \& $b$ \implies $f(b) = 0$

By the Rolle's Thm $\Rightarrow \exists c \in (a, b)$ such that $f'(c) = 0$.

where $f(x) = 7 + 4\sin(x)$

$f'(c) = 7 + 4\sin(c) = 0$

$\sin(c) = -\frac{7}{4} \leq -1$

$\implies \text{No such } c \text{ exists}$.

Contradicts the Rolle's Thm.

$\implies f(x)$ has at most one real sol'n.
\( f(x) = 7x - 4 \cos(x) + 10 \)

\( f(0) = 7(0) - 4\cos(0) + 10 = 6 > 0 \)

\( f(-\pi) = 7(-\pi) - 4\cos(-\pi) + 10 \\
= -7\pi + 14 < 0 \)

By the IVT \( \Rightarrow \exists c \in (-\pi, 0) \) such that \( f(c) = 0 \).

\( f(x) \) has at least one real root.

\( 7x - 4\cos x + 10 = 0 \) has exactly one real solution.
\[
\lim_{x \to \infty} \frac{\ln^2}{1 + \ln x} = \infty \Rightarrow \text{let } y = x
\]

\[
\Rightarrow \ln y = \ln x = \frac{\ln^2}{1 + \ln x}.
\]

\[
\lim_{x \to \infty} \frac{\ln^2 - \frac{1}{x}}{\frac{1}{x}} = \ln 2,
\]

\[
\lim_{x \to \infty} \frac{\ln^2}{1 + \ln x} = e^{\ln 2} = 2
\]

\[
\lim_{x \to 0} \frac{\sin (3x) - 3x + x^2}{\sin x \cdot \sin (2x)} = 0
\]

\[
\lim_{x \to \infty} \frac{3 \cos (3x) - 3 + 2x}{\cos x \cdot \sin (2x) + 2 \cos (2x) \sin x} = 0
\]

\[
\lim_{x \to 0} \frac{-9 \sin (3x) + 2}{-\sin x \sin (2x) + 2 \cos (2x) \cdot \cos x + 4 \sin (2x) \cdot \sin x + 2 \cos x \cdot \cos 2x} = \frac{1}{2}
\]
4.9 cont.

Table #2 on page 352

\[ f(x) = \cos x - \frac{4}{1+x^2} + \frac{10}{\sqrt{1-x^2}} + 7e^x \]

\[ F(x) = \sin x - 4\tan^{-1}x + 10\sin^{-1}x + 7e^x + C \]

---

Chapter 5

Integrals

5.1: Area under \( y = f(x) \) for \( a \leq x \leq b \)

\[ f(x_0), f(x_n) \]

\[ f(x_0) = f(a) \]

\[ f(x_n) = f(b) \]

\[ \Delta x = \frac{b-a}{n} \]

\[ x_1 = a + \Delta x \]

\[ x_2 = a + 2\Delta x \]

\[ \vdots \]

\[ x_i = a + i\Delta x \]
Area \approx \int_{x_i}^{x_i} f(x_i) \Delta x + f(x_i) \Delta x + \cdots + f(x_n) \Delta x = \text{RHS} \\
\text{RHS} = \sum_{i=1}^{n} f(x_i) \Delta x \\
Area = \int_{x_0}^{x_n} f(x_i) \Delta x + f(x_n) \Delta x + \cdots + f(x_n) \Delta x = \text{LHS} \\
\text{LHS} = \sum_{i=0}^{n-1} f(x_i) \Delta x \\
To \text{ approximate: } A = \frac{\text{RHS} + \text{LHS}}{2} \\
To \text{ find \ exact \ area: } = \lim_{n \to \infty} \text{RHS} \\
\text{Exact \ area \ under \ f(x) \ for \ } a \leq x \leq b, \\
A = \lim_{n \to \infty} \text{RHS} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x \\
\text{where } \Delta x = \frac{b-a}{n} \text{ and } x_i = a + i \Delta x, \\
\text{Riemann \ Sum}
1. \[ \sum_{i=1}^{n} k = nk \]

2. \[ 1 + 2 + 3 + \ldots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \]

3. \[ 1^2 + 2^2 + 3^2 + \ldots + n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \]

4. \[ 1^3 + 2^3 + 3^3 + \ldots + n^3 = \sum_{i=1}^{n} i^3 = \left[ \frac{n(n+1)}{2} \right]^2 \]
Ex: Find the exact area
under \( f(x) = x^2 + 3x - 5 \) for \( 0 \leq x \leq 2 \).

\[
\text{Area} = \lim_{n \to \infty} \text{RHS} = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x
\]

where \( \Delta x = \frac{b-a}{n} = \frac{2-0}{n} = \frac{2}{n} \)

and \( x_i = a + i \Delta x = 0 + i \cdot \frac{2}{n} = \frac{2i}{n} \)

\[
= \lim_{n \to \infty} \sum_{i=1}^{n} f\left( \frac{2i}{n} \right) \cdot \frac{2}{n}
\]

\[
= \lim_{n \to \infty} \sum_{i=1}^{n} \left[ \left( \frac{2i}{n} \right)^2 + 3 \left( \frac{2i}{n} \right) - 5 \right] \cdot \frac{2}{n}
\]

\[
= \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{4i^2}{n^2} + \frac{6i}{n} - 5 \right) \cdot \frac{2}{n}
\]

\[
= \lim_{n \to \infty} \left[ \frac{4}{n^2} \left( \sum_{i=1}^{n} i^2 \right) + \frac{6}{n} \left( \sum_{i=1}^{n} i \right) - 5n \right] \cdot \frac{2}{n}
\]

\[
= \lim_{n \to \infty} \left[ \frac{4}{n^2} \left( \frac{n(n+1)(2n+1)}{6} \right) + \frac{6}{n} \left( \frac{n(n+1)}{2} \right) - 5n \right] \cdot \frac{2}{n}
\]

\[
= \frac{8.2}{6} + \frac{6.2}{2} - 10 = \#
\]
Exam #3, Cover Chapter 4.

1. Use the Rolle's Theorem to prove

2. Find extreme value of \( f(x) \) over \([a, b]\)

3. Analyze \( f(x) \)

4. \( \perp \parallel \)

5. Optimization Choose 2 out of 3.
   a) \( \Delta \)
   b) \( \triangleleft \)
   c) Maximize Revenue/Profit

6. Newton's method \( \sqrt{35} \)

7. Find Anti-derivative

8. Find \( y \) given \( y'' = \frac{1}{x} \)
   \( y'(0) = 1 \)
   \( y(1) = -5 \)

9. \( \lim_{x \to 0} \frac{2}{x} \)