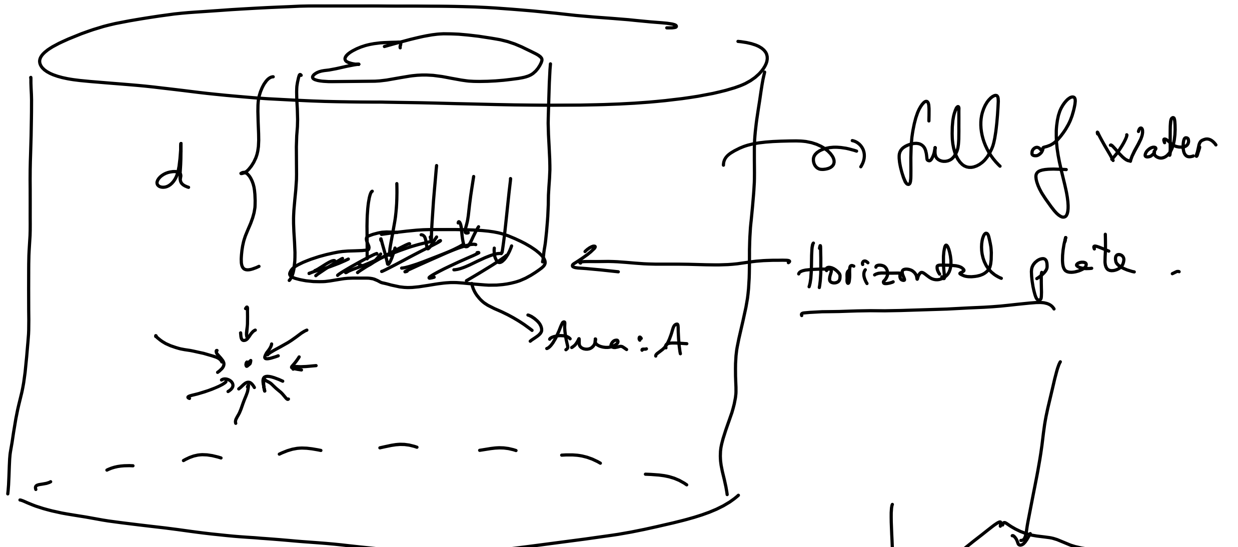
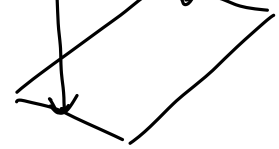
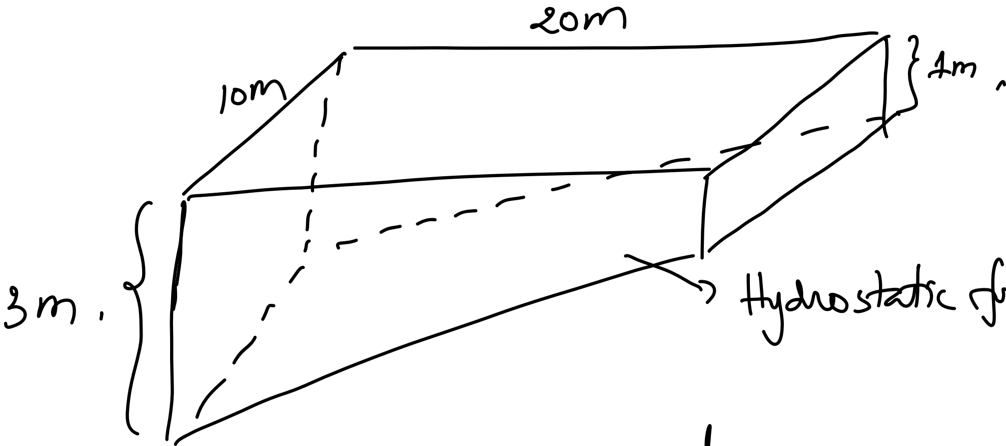




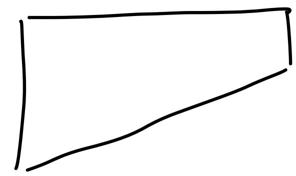
$$\text{Volume} = A \cdot d = (\text{Area})(\text{depth})$$

$$\text{Force} = \left\{ \begin{matrix} 9800\ N \\ \text{or} \\ 62.5\ \text{lbs} \end{matrix} \right\} \cdot A d$$

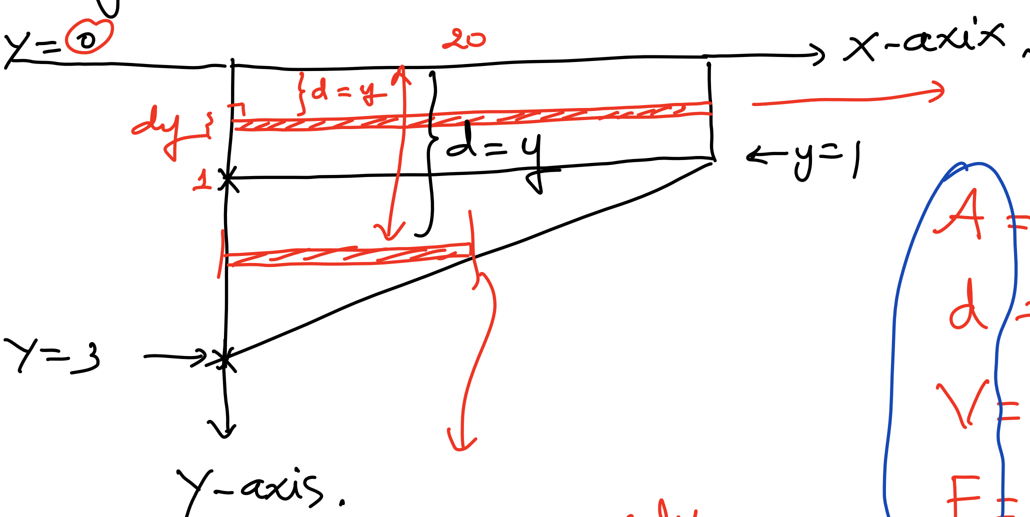
Ex1: A swimming pool is 20 m long and 10 m wide. The bottom is flat (but not horizontal) and the sides are vertical. The water is 3m deep at one end and 1 m deep at the other end. Find the force of the water on one of the sides.

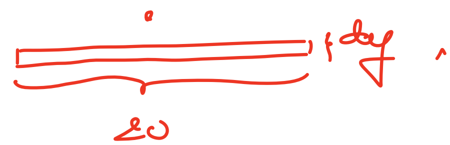


Hydrostatic force to



⇒ Put the wall onto xy-coord.





$A = 20\,dy$

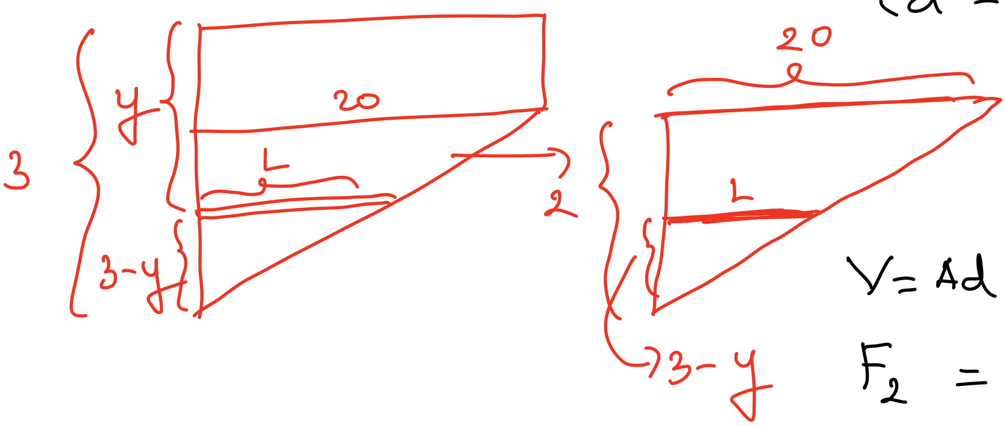
$d = y$

$V_1 = Ad = 20y\,dy$

$F_1 = \int_0^1 9800 \cdot 20 \cdot y\,dy$

$L = 10(3-y)$

$\Rightarrow \begin{cases} A = 10(3-y)\,dy \\ d = y \end{cases}$



Similar Δ:

$\frac{L}{20} = \frac{3-y}{2} \Rightarrow L = 10(3-y)$

$V = Ad = 10(3-y) \cdot y\,dy$

$F_2 = \int_1^3 9800 \cdot 10\,(3-y) \cdot y\,dy.$

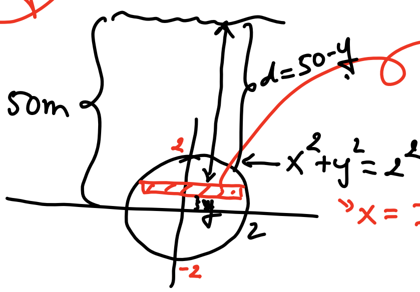
Total hydrostatic force: $F = F_1 + F_2 = 9800(20)\int_0^1 y\,dy + 98000 \int_1^3 (3y - y^2)\,dy$

$= 9800\left[10y \Big|_0^1 + 10\left(\frac{3}{2}y^2 - \frac{1}{3}y^3\right)\Big|_1^3 \right] = 9800\left(10 + 10\left(\frac{27}{2} - 9 - \frac{3}{2} + \frac{1}{3}\right)\right)$

$= \boxed{\# N.}$

Ex2: Determine the hydrostatic force on a vertical circular gate of radius 2m, which is under the water.

a) 50 m from its center.



$\Rightarrow x = \pm\sqrt{4-y^2}$

$\underbrace{\qquad}_{L} \; dy$

$L = x_r - x_L = \sqrt{4-y^2} - (-\sqrt{4-y^2}) = 2\sqrt{4-y^2}$

$A = 2\sqrt{4-y^2}\,dy$ ✓

$d = 50-y$

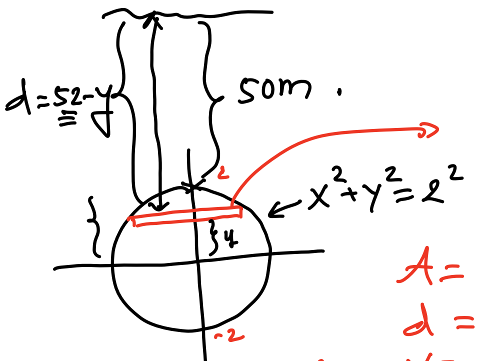
$V = Ad = (50-y)\cdot 2\sqrt{4-y^2}\,dy$

$F = 9800\cdot 2\cdot\int_{-2}^{2}(50-y)\cdot\sqrt{4-y^2}\,dy$

$= 19{,}600\left[50\underbrace{\int_{-2}^{2}\sqrt{4-y^2}\,dy}_{} - \cancel{\int_{-2}^{2} y\sqrt{4-y^2}\,dy}\right]$ → 0

$\left(19{,}600\cdot 50\cdot\frac{1}{2}\cdot\pi\cdot 2^2\right) = \# N.$

b) 50 m from its top.



$\underbrace{\qquad}_{L = 2\sqrt{4-y^2}} \; dy$

$A = 2\sqrt{4-y^2}\,dy$

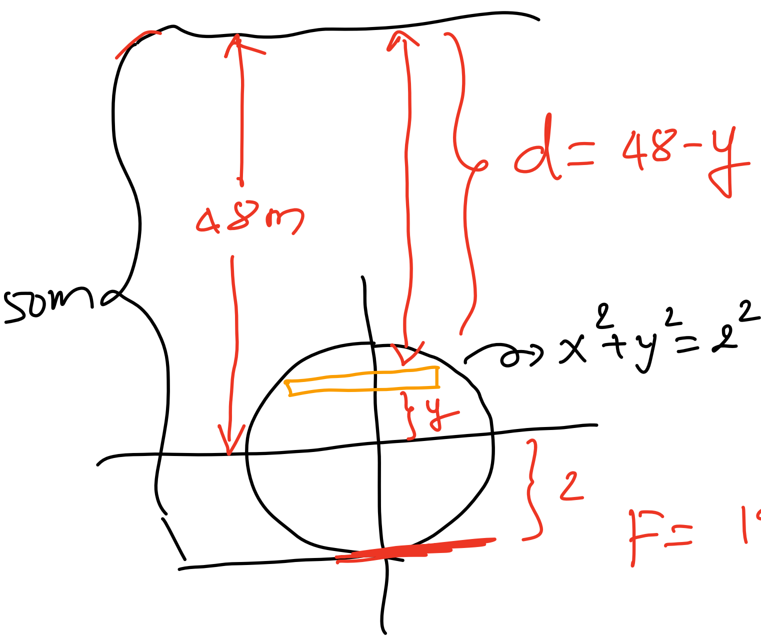
$d = 52-y$

$V = 2\cdot\sqrt{4-y^2}\,(52-y)\,dy$

$F = 9800\cdot 2\int_{-2}^{2}\sqrt{4-y^2}\,(52-y)\,dy = 19{,}600\left[52\int_{-2}^{2}\sqrt{4-y^2}\,dy - \cancel{\int_{-2}^{2} y\sqrt{4-y^2}\,dy}\right]$ → 0

$= \left[19{,}600\cdot 52\cdot\frac{1}{2}\cdot\pi\cdot 2^2\right] = \# N.$

c) 50 m from its bottom.



$\underbrace{\qquad}_{L = 2\sqrt{4-y^2}} \; dy$

$A = 2\sqrt{4-y^2}\cdot dy$

$d = 48-y$

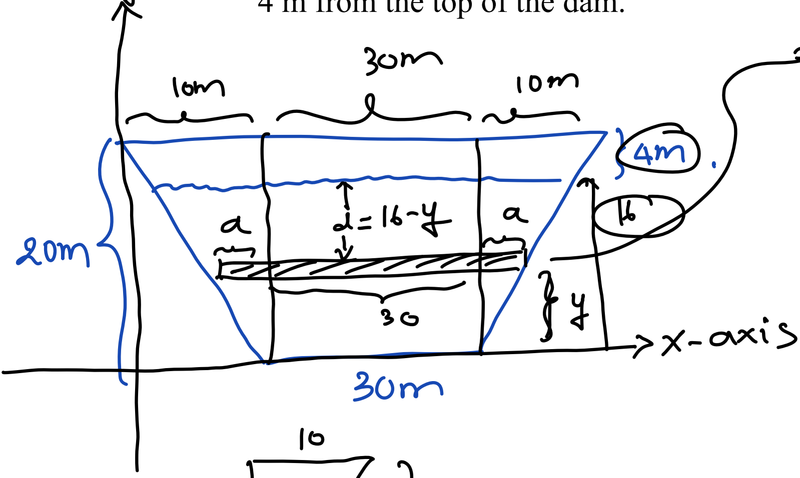
$V = 2\sqrt{4-y^2}\,(48-y)\,dy$

$F = \int_{-2}^{2} 9800\cdot 2\sqrt{4-y^2}\,(48-y)\,dy$

$F = 19{,}600\left[48\underbrace{\int_{-2}^{2}\sqrt{4-y^2}\,dy}_{\frac{1}{2}\pi\cdot r^2} - \cancel{\int_{-2}^{2} y\sqrt{4-y^2}\,dy}\right]$ → 0

$= 19{,}600\cdot 48\cdot\frac{1}{2}\pi(4) = \dots = \# N.$

Ex3: A dam has the shape of a trapezoid. The height is 20 m, and the width is 50 m at the top and 30 m at the bottom. Find the force on the dam due to hydrostatic pressure if the water level is 4 m from the top of the dam.



$$\underbrace{\rule{3cm}{0.4pt}}_{L}\ \} dy$$

$$L = 30 + 2a = 30 + 2\left(\tfrac{1}{2}y\right) = 30 + y$$

$$A = (30+y)\,dy$$

$$d = 16 - y$$

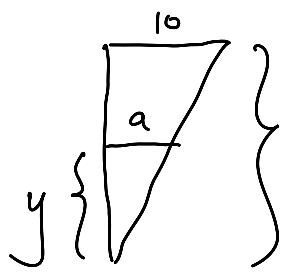
$$V = Ad = (30+y)(16-y)\,dy$$

$$F = 9800\int_0^{16} (30+y)(16-y)\,dy$$

$$= 9800\int_0^{16} (480 - 14y - y^2)\,dy$$

$$= 9800\left[480(16) - 7(16)^2 - \tfrac{1}{3}(16)^3\right]$$

$$= \dots = \#\ N.$$

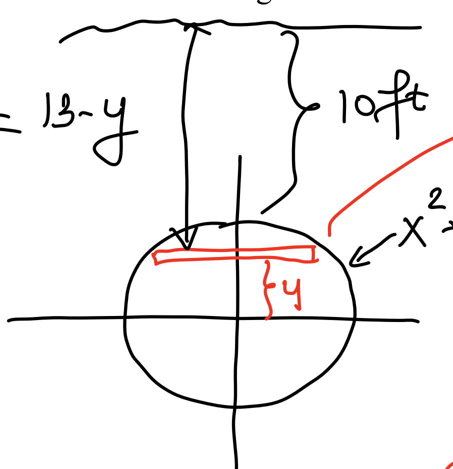


$$20 \Rightarrow \frac{a}{10} = \frac{y}{20}$$

$$a = \tfrac{1}{2}y$$

Ex4: Find the hydrostatic force on one end of a cylindrical drum with radius 3 ft if the drum is submerged in water 10 ft from its top.

$$d = 13 - y$$



$$x^2 + y^2 = 3^2$$

$$x = \pm\sqrt{9 - y^2}$$

$$\underbrace{\rule{3cm}{0.4pt}}_{L}\ dy$$

$$L = 2\sqrt{9-y^2}$$

$$A = 2\sqrt{9-y^2}\cdot dy$$

$$d = 13 - y$$

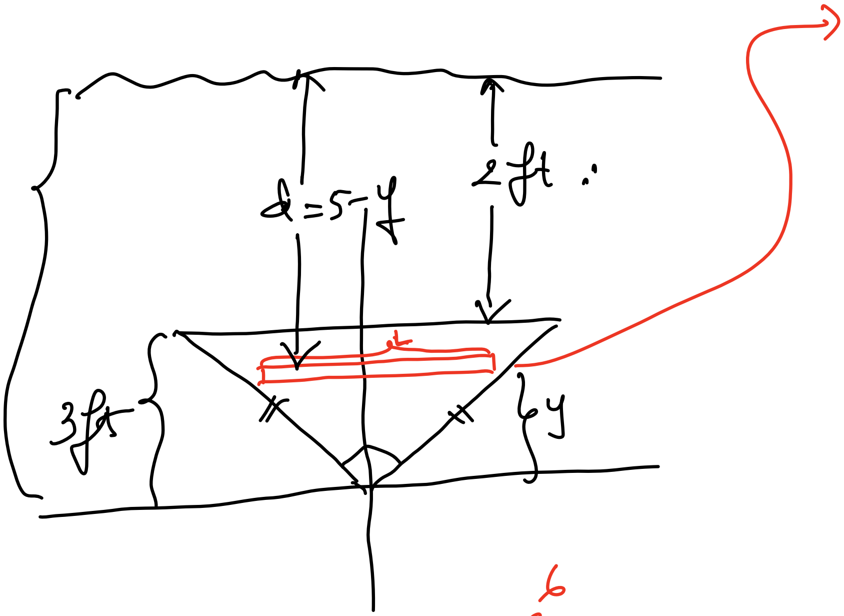
$$V = 2\sqrt{9-y^2}\,(13-y)\,dy$$

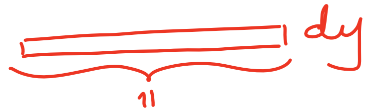
$$F = \underbrace{(62.5)\cdot 2}\int_{-3}^{3} \sqrt{9-y^2}\,(13-y)\,dy$$

$$= 125\left[13\underbrace{\int_{-3}^{3}\sqrt{9-y^2}\,dy}_{\frac{1}{2}\pi\cdot r^2} - \cancel{\int_{-3}^{3} y\sqrt{9-y^2}\,dy}^{\,0}\right]$$

$$= (125)(13)\cdot\tfrac{1}{2}\pi\cdot(3)^2 = \dots = \#\ lbs.$$

Ex5: A flat isosceles right triangular plate with base 6 ft and height 3 ft is submerged vertically, base up, 2 ft below the surface of a swimming pool. Find the force exerted by the water against one side of the plate.





$L = 2y$

$A = 2y\,dy$

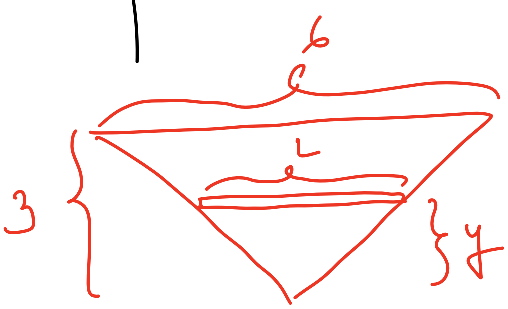
$d = 5 - y$

$V = 2y(5-y)\,dy$

$F = (62.5)(2)\int_0^3 (5y - y^2)\,dy$

$= 125\left[\frac{5}{2}(3)^2 - \frac{1}{3}(3)^3\right]$

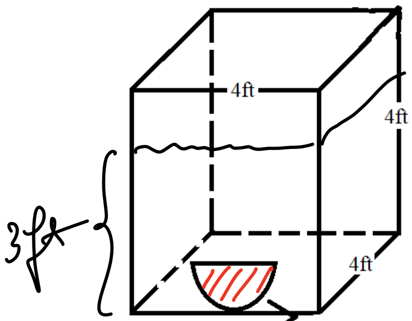
$= \dots = \#\ \text{lbs}.$



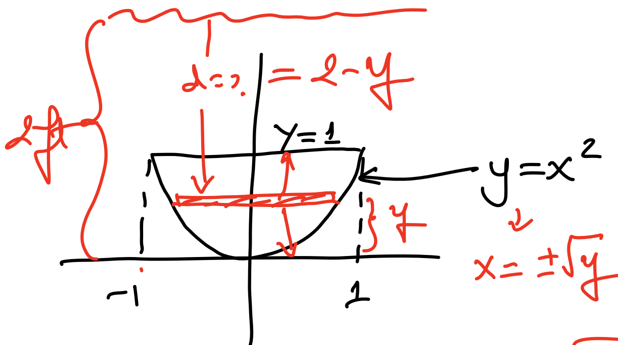
$\frac{L}{6} = \frac{y}{3} \Rightarrow L = 2y.$

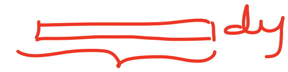
Ex6: The cubical metal tank shown here has a parabolic gate, held in place by bolts and designed to withstand a fluid force of 160 lb without rupturing. The liquid you plan to store has a weight density of 50 lb/ft^3.

a) What is the fluid force on the gate when the liquid is 2 ft deep?



$y = x^2 \;\; for \; -1 \le x \le 1$





$L = \sqrt{y} - (-\sqrt{y})$
$= 2\sqrt{y}.$

$A = 2\sqrt{y}\cdot dy.$

$d = 2-y.$

$V = 2\sqrt{y}\,(2-y)\,dy.$

$F = \int_0^1 50 \cdot 2\sqrt{y}\,(2-y)\,dy.$

$100\int_0^1 \left(2y^{1/2} - y^{3/2}\right) dy$

$= 100\cdot\left[2\,\frac{2}{3}y^{3/2} - \frac{2}{5}y^{5/2}\right]_0^1$

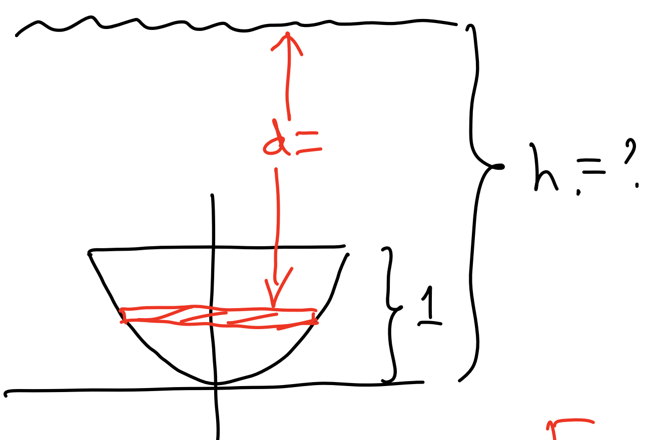
$= 100\left[\frac{4}{3} - \frac{2}{5}\right]$

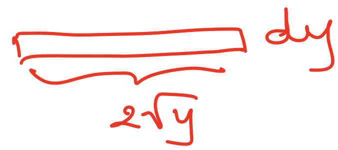
$= 100\left[\frac{20-6}{15}\right] = \frac{100(14)}{15}$

$= 93.33 \text{ lb}.$

---

b) What is the maximum height to which the container can be filled without exceeding its design limitation?





$A = 2\sqrt{y}\,dy.$

$d = h-y$

$V = 2\sqrt{y}\,(h-y)\,dy.$

$F = \int_0^1 50 \cdot 2 \cdot \sqrt{y}\,(h-y)\,dy.$

$\Rightarrow F = 100\int_0^1 \left(hy^{1/2} - y^{3/2}\right) dy = 100\left[h\cdot\frac{2}{3}y^{3/2} - \frac{2}{5}y^{5/2}\right]_0^1$

$= \frac{\cancel{100}\left[\frac{2h}{3} - \frac{2}{5}\right]}{100} \overset{\text{set}}{=} \frac{\cancel{160}}{100}.$

$$\frac{2h}{3} - \frac{2}{5} = 1.6$$

$$\Rightarrow h = \frac{3}{2}\left(1.6 + \frac{2}{5}\right) = 3 \text{ ft}.$$

faculty.mtsac.edu/ftran