Section 7.5 *Strategy for Integration*

1. Integration techniques:
   a) Substitution
   b) Parts
   c) Trig. Integrals
   d) Trig. Subs
   e) Partial Fraction
   f) Others. $\leftarrow$

$\cancel{x = u}$,

Ex: Integrate the following:

a) $\int \sqrt{2+\sqrt{3x+1}}\,dx$

Nested root $\Rightarrow$ $u = \sqrt{2+\sqrt{3x+1}}$

$$u^2 = 2 + \sqrt{3x+1}$$

$$(u^2-2)^2 = \left(\sqrt{3x+1}\right)^2$$

$$u^4 - 4u^2 + 4 = 3x+1$$

$$(4u^3 - 8u)\,du = 3\,dx \quad . . .$$

b) $\int e^{\sqrt[4]{x}}\,dx$ $\left\{ \begin{array}{l} u = \sqrt[4]{x} \ ; \ u^4 = x \\ 4u^3\,du = dx \end{array} \right.$

$4\int e^u u^3\,du \longleftarrow$ parts.

c) $\int \frac{\sin(3x)}{\cos^2(3x)+4\cos(3x)-21}dx$

$$\begin{cases} u = \cos(3x) \\ du = -3\sin(3x)dx \\ -\frac{1}{3}du = \sin(3x)dx \end{cases}$$

$$= -\frac{1}{3}\int \frac{du}{u^2+4u-21} = -\frac{1}{3}\int \frac{du}{(u+7)(u-3)}$$

partial fraction.

d) $\int \frac{2x-3}{\left[4x^2+4x-15\right]^{3/2}}dx \rightarrow$

$$4x^2+4x-15 = \underbrace{(2x)^2+2(4x)+1}_{} - 16$$

$$= (2x+1)^2 - 4^2$$

$\Rightarrow$ Trig. Sub: Let $2x+1 = 2\sec\theta$

e) $\int \frac{dx}{1-\cos x}$ multiply top & bot. by the conjugate.

$$\int \frac{1}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x}\cdot dx = \int \frac{1+\cos x}{1-\cos^2 x}\,dx$$

$$= \int \frac{1+\cos x}{\sin^2 x}\,dx = \int \left(\csc^2 x + \cot x\cdot\csc x\right)dx$$

$$= -\cot x - \csc x + C.$$

f) $\int \frac{dx}{\sqrt{x}+4\sqrt[3]{x}}$ $\rightsquigarrow$ let $x = u^6$

$dx = 6u^5\,du$.

$$= \int \frac{6u^5\,du}{\sqrt{u^6}+4\sqrt[3]{u^6}} = 6\int \frac{u^5}{u^3+4u^2}\,du$$

$$= 6\int \frac{u^3}{u+4}\,du \leftarrow \text{Synthetic Div.}$$

g) $\int \sin^{-1}(5x)\,dx$ $= x\sin^{-1}(5x) - \int \frac{5x}{\sqrt{1-25x^2}}\,dx$

↖ u-sub.

$\sin^{-1}(5x)$	dx
$\frac{5}{\sqrt{1-25x^2}}\,dx$	x

$$-\int$$

h) $\int \frac{2x^3 - 2x^2 - 25x - 31}{x^2 - x - 12} dx = \int \left[2x - \frac{x+31}{(x+3)(x-4)} \right] dx$

partial fraction.

$$\begin{array}{r|l} & 2x \\ \hline 1x^2 - x - 12 & 2x^3 - 2x^2 - 25x - 31 \\ & 2x^3 - 2x^2 - 24x \\ \hline & \qquad\qquad -x - 31 \end{array}$$

Remainder

i) $\int \ln(4x+5)\, dx = \underbrace{x \ln(4x+5)}_{A} - \int \frac{4x+5-5}{4x+5} dx$

$=1$

$\ln(4x+5)$	dx
$\frac{4}{4x+5} dx$	x

$= A - \int \left(1 - \frac{5}{4x+5}\right) dx.$

$= A - \left[x - \frac{5}{4} \ln|4x+5| \right] + C.$

j) $\int (7x^3 - 5x + 3) \sin(2x)\, dx$

$7x^3 - 5x + 3$	(+) $\sin(2x) dx$
$21x^2 - 5$	(−) $-\frac{1}{2}\cos(2x)$
$42x$	(+) $-\frac{1}{4}\sin(2x)$
42	(−) $\frac{1}{8}\cos(2x)$
0	$\frac{1}{16}\sin(2x)$

$\int \cos(kx)\, dx$

$\parallel$

$\frac{1}{k}\sin(kx) + C$

k) $\int\sqrt{\frac{1-x}{1+x}}dx = \int \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}}\, dx = \int \frac{1-x}{\sqrt{1-x^2}}dx.$

$$= \underbrace{\int \frac{1}{\sqrt{1-x^2}}dx}_{\sin^{-1}x} - \int \frac{x\,dx}{\sqrt{1-x^2}}$$

let $u=\sqrt{1-x^2}$

$u^2 = 1-x^2$

$2u\,du = -2x\,dx$

$-u\,du = x\,dx$

$$= \sin^{-1}x + \int \frac{\cancel{u}\,du}{\cancel{u}}$$

$$= \sin^{-1}x + \sqrt{1-x^2} + C.$$

l) $\int x^8 \sqrt[4]{2x^3+1}\,dx \to \int x^6 \cdot \sqrt[4]{2x^3+1} \cdot x^2dx$

let $u = \sqrt[4]{2x^3+1}$

$u^4 = 2x^3+1$

$4u^3du = 6x^2dx.$

$\frac{2}{3}u^3du = x^2dx$