Section 6.3 ***Volumes by Cylindrical Shells***

There is another way to find volumes of solids of rotation that can be useful when the axis of revolution is perpendicular to the axis containing the natural interval of integration. Instead of summing volumes of thin slices, we sum volumes of thin cylindrical shells that grow outward from the axis of revolution like three rings.

$$V = \int_a^b 2\pi(radius)\begin{pmatrix} shell \\ height \end{pmatrix} dx$$

<u>***Ex***</u>: The region enclosed by the x-axis and the parabola $y = f(x) = 3x - x^2$ is revolved about the line $x = -1$ to generate the shape of a solid as follows. What is the volume of the solid?

Ex: The region bounded by the following curves is rotated about the indicated line. Sketch and then setup integral(s) for their volumes:

a) $y = x^2 - x - 6$ *and* $y = 2x + 4$ is rotated about the line $x = 6$.

b) $x = -y^2 + 2y + 3$ *and* $x = y - 9$ is rotated about the line $y = 4$

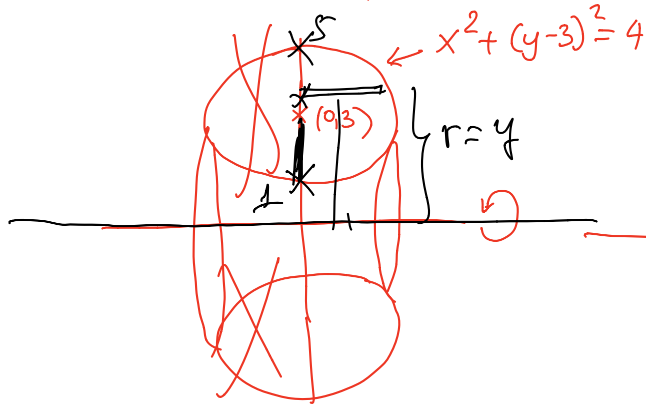
Note: $\left\{ \text{disc} / \text{washer} \right\} \to \perp$ to the axis of rotation.

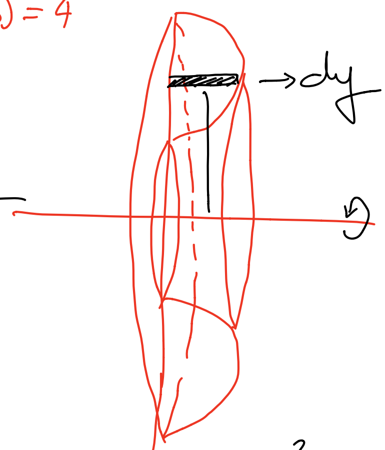
$\{$shell method$\} \to$ rectangle must be parallel to the axis of rotation.

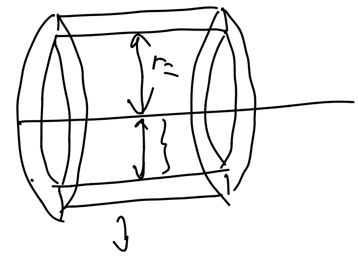
c) $R=\{(x,y)\in R^2 \mid x^2+(y-3)^2\le 4\}$ is rotated about the x – axis.

$x^2+(y-3)^2=4=2^2$

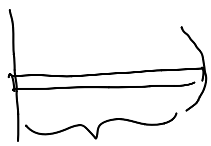
$x^2+y^2=2^2$







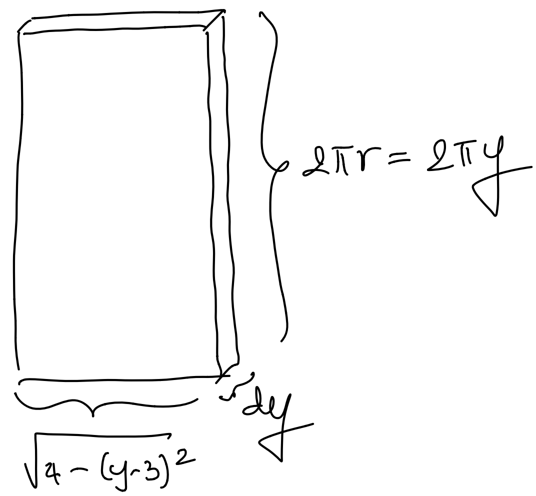
r=



$x^2+(y-3)^2=4$

$x=\sqrt{4-(y-3)^2}$

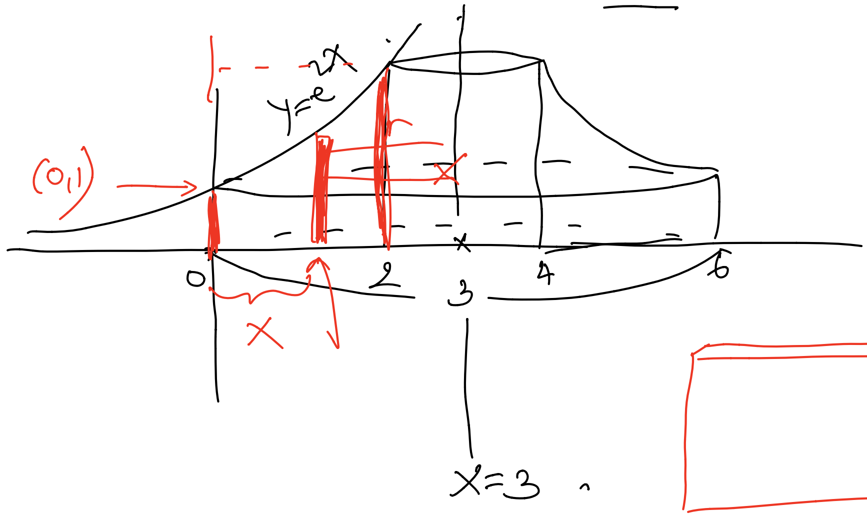
$x_r - x_L = \sqrt{4-(y-3)^2}-0$

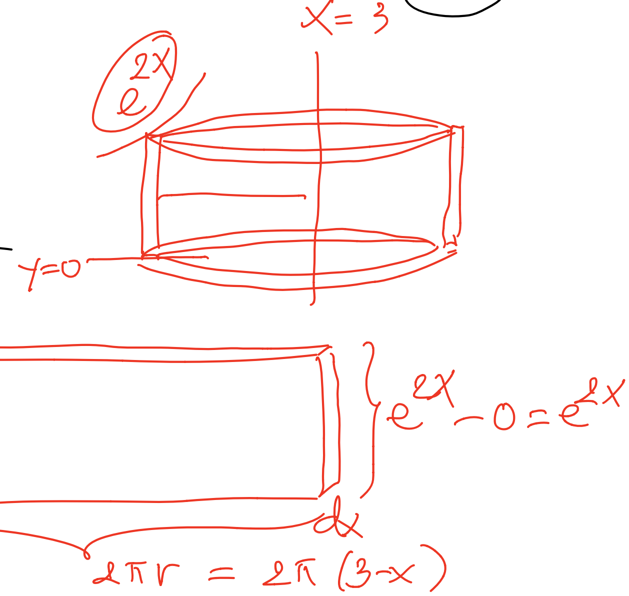


$$V=2\int_1^5 2\pi y\cdot\sqrt{4-(y-3)^2}\cdot dy$$

Ex: Sketch and then setup integral(s) for volume:

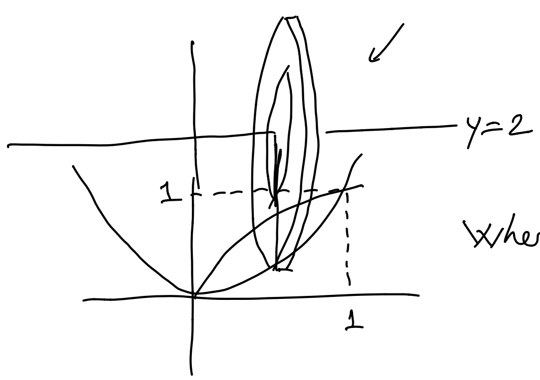
a) The region bounded by $y = e^{2x}; y = 0 \;\; for \;\; 0 \le x \le 2$ is rotated about the line x = 3.





$$V = 2\pi \int_0^2 (3-x)e^{2x}\,dx$$

b) The region bounded by $y = \sqrt{x} \;\; and \;\; y = x^2$ is rotated about the line y=2.

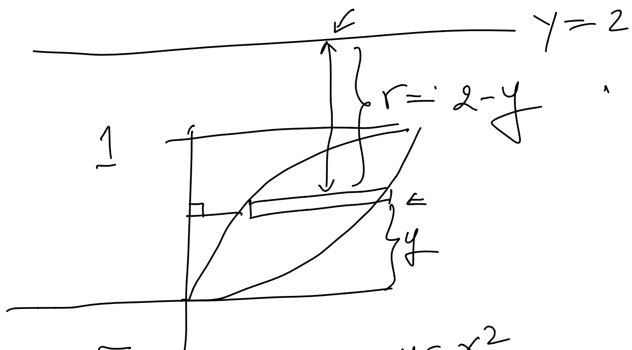


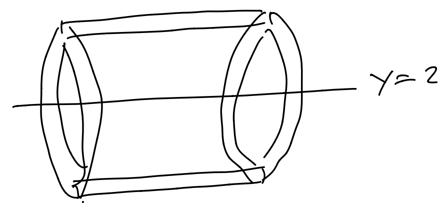
$$V = \pi\left(R_o^2 - r_i^2\right)dx$$

Where: $R_o = $ (from $y=x^2$ to $y=2$) $= y_{top} - y_{bot} = 2 - x^2$

$r_i = $ (from $y=\sqrt{2}$ to $y=2$) $= y_{top} - y_{bot} = 2 - \sqrt{x}$

$$\Rightarrow \quad V = \pi \int_0^1 \left[(2-x)^2 - (2-\sqrt{2})^2\right] dx$$





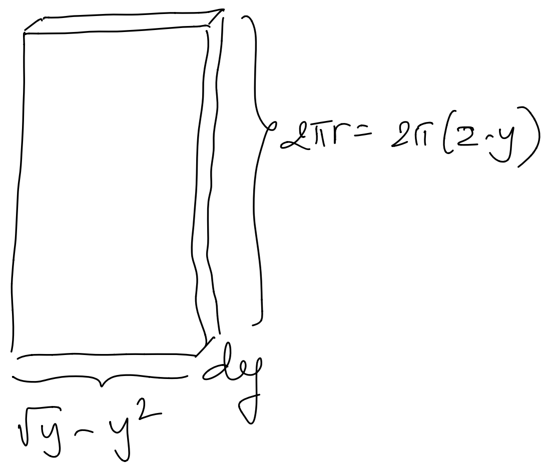
$y = \sqrt{x}$

$x_L = y^2$

$\leftarrow y = x^2$

$x_r = \sqrt{y}$

$x_r - x_L = \sqrt{y} - y^2$



$$V = 2\pi \int_0^1 \left(\sqrt{y} - y^2\right)(2-y)\,dy$$