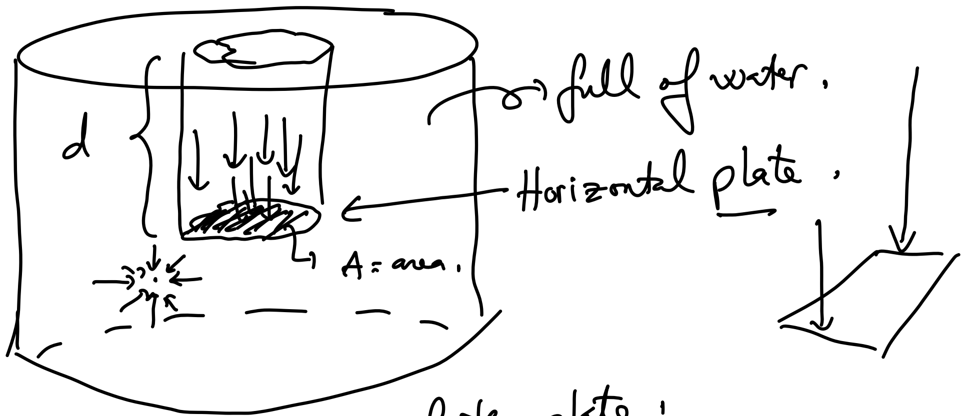
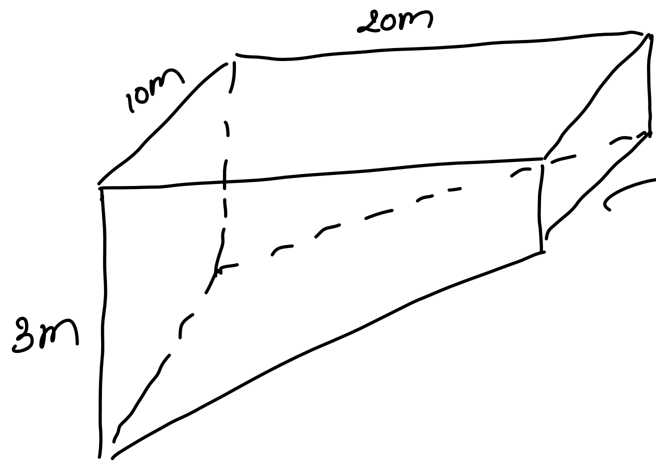


Find the volume on top of the plate:

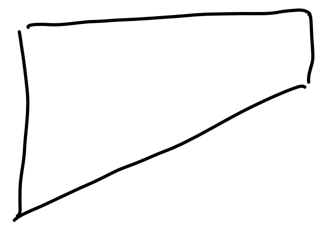
$$V = A \cdot d$$

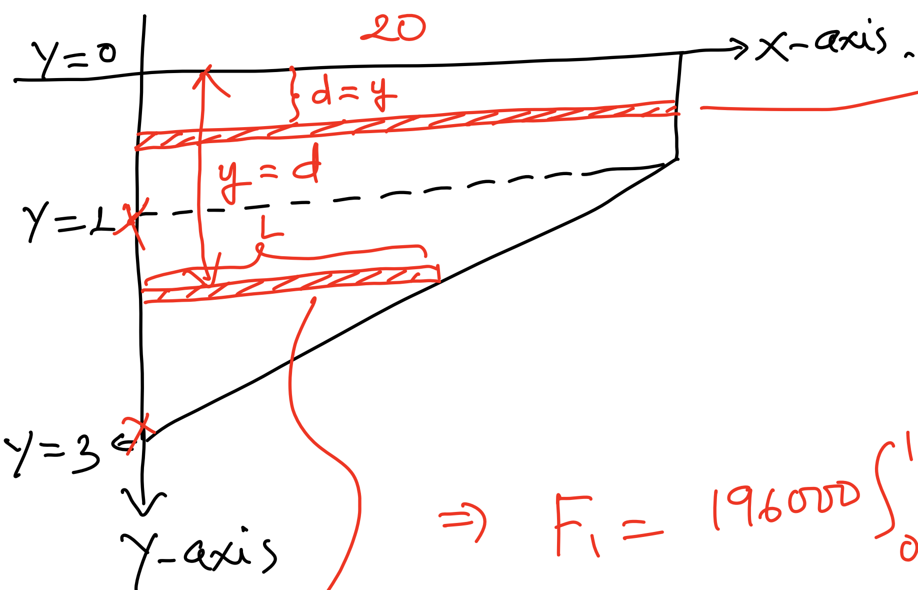
Force $$F = \begin{Bmatrix} 9800\,N \\ \text{or} \\ 62.5\,lb \end{Bmatrix} \cdot \text{Volume} = \begin{cases} 9800\,N \cdot Ad & \text{if measure in m} \\ \quad \text{or} & \\ 62.5\,lb \cdot Ad & \text{if — ft.} \end{cases}$$

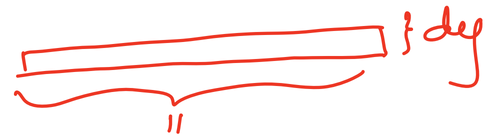
Ex1: A swimming pool is 20 m long and 10 m wide. The bottom is flat (but not horizontal) and the sides are vertical. The water is 3m deep at one end and 1 m deep at the other end. Find the force of the water on one of the sides.



Hydrostatic force on







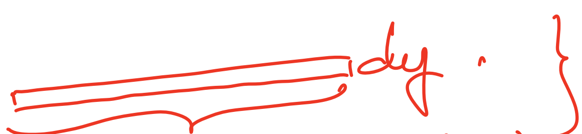
$A = 20\,dy$

$d = y$

$V = Ad = 20y\,dy$

$F_1 = \int_0^1 9800 \cdot 20y \cdot dy$

$\Rightarrow F_1 = 196000 \int_0^1 y\,dy = \frac{196000}{2}(1)^2 = \underline{98000\ N}$



$A = 10(3-y)\,dy$

$d = y$

$V = Ad = 10y(3-y)\,dy$

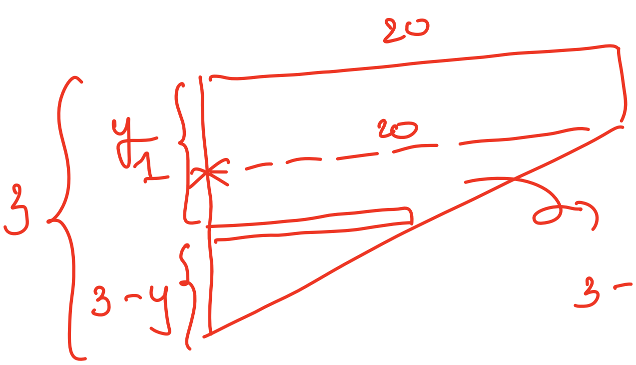
$F_2 = \int_1^3 9800 \cdot 10y(3-y)\,dy = 98000 \int_1^3 (3y - y^2)\,dy$

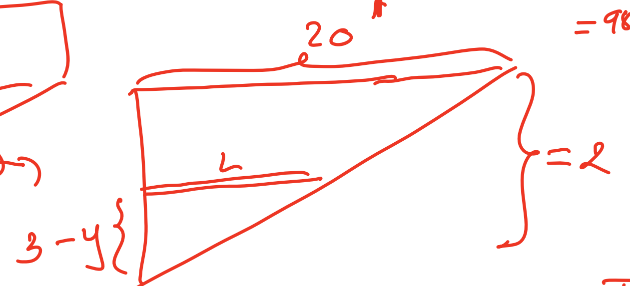
$= 98000\left[\frac{3}{2}y^2 - \frac{1}{3}y^3\right]_1^3$

$= 98000\left[\frac{27}{2} - 9 - \frac{3}{2} + \frac{1}{3}\right]$

$= 98000\left[\frac{81-54-9+2}{6}\right]$

$= 326{,}666.67\ N.$





Similar Δ: $\frac{L}{20} = \frac{3-y}{2} \Rightarrow L = 10(3-y)$

Total hydrostatic force:

$F = F_1 + F_2$

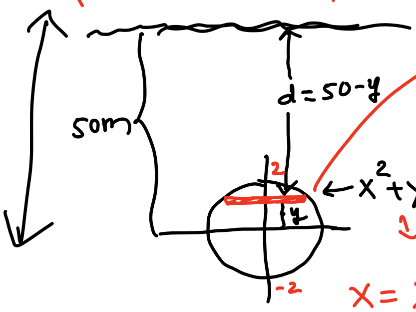
$= \underline{98000 + 326{,}666.67}$

$\#N.$

circular

~~Ex2~~: Determine the hydrostatic force on a vertical gate of radius 2m, which is under the water.

a) 50 m from its center.



$$x=\pm\sqrt{4-y^2}$$

$$L = ? = x_r - x_L = \sqrt{4-y^2} - \left(-\sqrt{4-y^2}\right) = 2\sqrt{4-y^2}$$

$$A = 2\sqrt{4-y^2}\cdot dy$$

$$d = 50-y$$

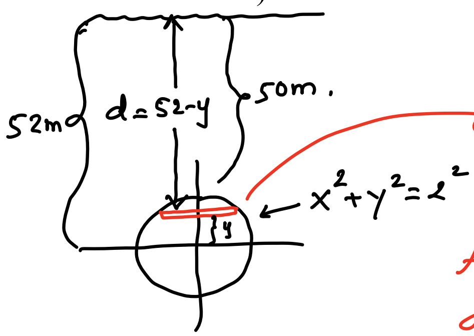
$$V = Ad = 2\sqrt{4-y^2}\,(50-y)\,dy$$

$$F = \underbrace{9800\cdot 2}\int_{-2}^{2}\sqrt{4-y^2}\,(50-y)\,dy$$

$$= 19{,}600\left[50\underbrace{\int_{-2}^{2}\sqrt{4-y^2}\,dy} - \underbrace{\cancel{\int_{-2}^{2} y\sqrt{4-y^2}\,dy}}_{\text{ok.?}}\right]$$

$$= \boxed{19{,}600\cdot 50\cdot \tfrac{1}{2}\pi\cdot(2)^2} = \dots = \#N$$

b) 50 m from its top.



$$L = 2\sqrt{4-y^2}$$

$$A = 2\sqrt{4-y^2}\,dy$$

$$d = 52-y$$

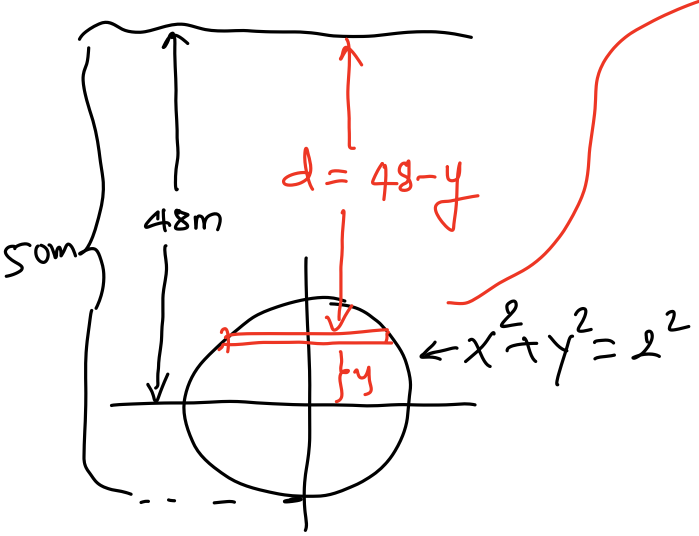
$$V = Ad = 2\sqrt{4-y^2}\,(52-y)\,dy$$

$$F = \underbrace{9800\cdot 2}\int_{-2}^{2}\sqrt{4-y^2}\,(52-y)\,dy$$

$$F = 19{,}600\left[52\underbrace{\int_{-2}^{2}\sqrt{4-y^2}\,dy}_{\frac{1}{2}\pi\cdot r^2} - \underbrace{\int_{-2}^{2} y\sqrt{4-y^2}\,dy}_{=0}\right]$$

$$= \boxed{19{,}600\cdot 52\cdot \tfrac{1}{2}\pi\cdot 2^2} = \dots = \#N.$$

c) 50 m from its bottom.



$$L = 2\sqrt{4-y^2}$$

$$A = 2\sqrt{4-y^2}\cdot dy$$

$$d = 48-y$$

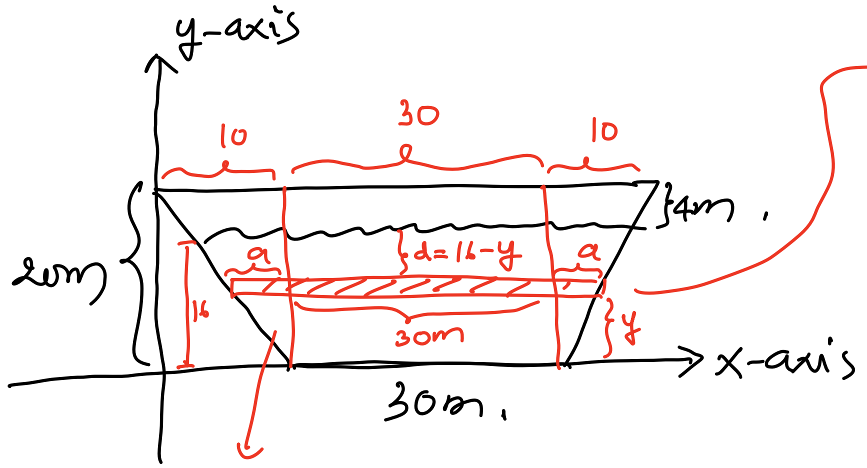
$$V = 2\sqrt{4-y^2}\,(48-y)\,dy$$

$$F = 9800\cdot 2\int_{-2}^{2}\sqrt{4-y^2}\,(48-y)\,dy$$

$$= F = 19{,}600\left[48\underbrace{\int_{-2}^{2}\sqrt{4-y^2}\,dy}_{\frac{1}{2}\pi\cdot r^2} - \cancel{\int_{-2}^{2} y\sqrt{4-y^2}\,dy}\to 0\right]$$

$$= 19{,}600\cdot 48\cdot \tfrac{1}{2}\pi(2)^2 = \dots = \#N$$

Ex3: A dam has the shape of a trapezoid. The height is 20 m, and the width is 50 m at the top and 30 m at the bottom. Find the force on the dam due to hydrostatic pressure if the water level is 4 m from the top of the dam.



$L = 30 + 2a = 30 + 2\left(\frac{1}{2}y\right)$

$L = 30 + y$

$A = (30+y)\,dy$

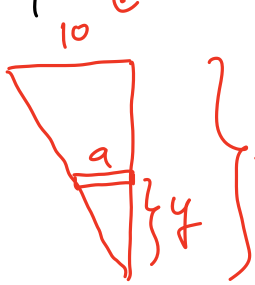
$d = 16 - y$

$V = Ad = (30+y)(16-y)\,dy$

$F = 9800 \int_0^{16} (480 - 14y - y^2)\,dy$

$= 9800 \cdot \left[480(16) - 7(16)^2 - \frac{1}{3}(16)^3\right]$

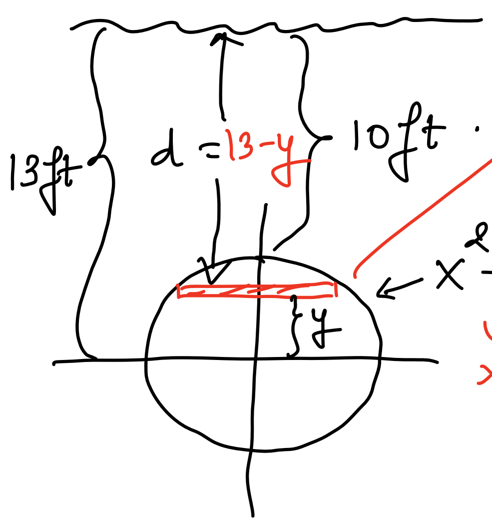
$= \cdots = \#\,N$



Similar Δ: $\frac{a}{10} = \frac{y}{20}$

$\Rightarrow a = \frac{1}{2}y$

Ex4: Find the hydrostatic force on one end of a cylindrical drum with radius 3 ft if the drum is submerged in water 10 ft from its top.



$x^2 + y^2 = 3^2$

$x = \pm\sqrt{9-y^2}$

$L = x_r - x_L = 2\sqrt{9-y^2}$

$A = 2\sqrt{9-y^2}\,dy$

$d = 13 - y$

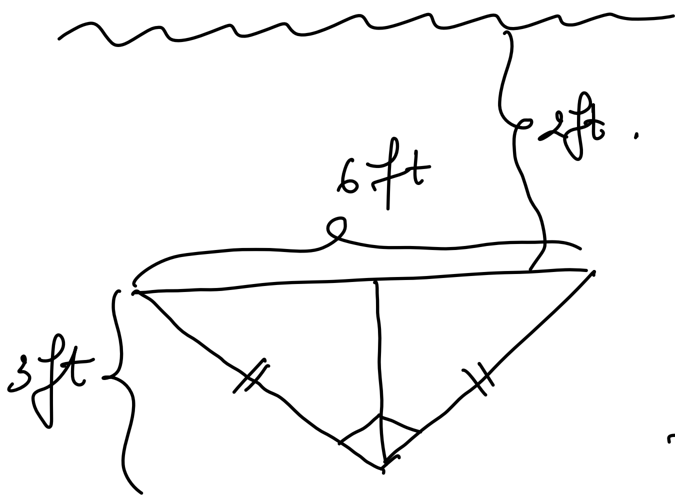
$V = 2\sqrt{9-y^2}\,(13-y)\,dy$

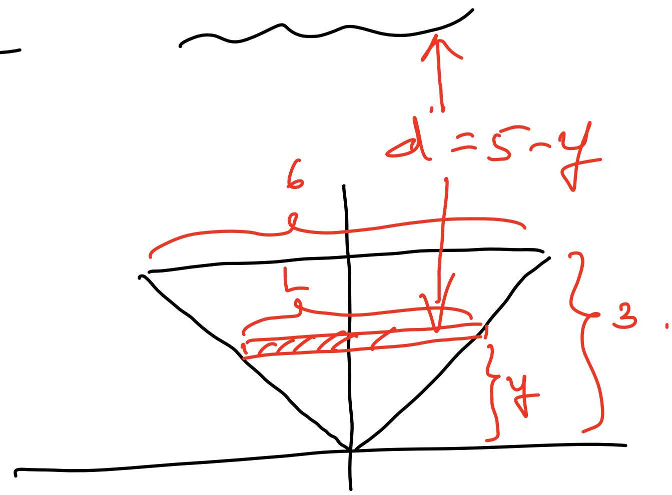
$F = \int_{-3}^{3} (62.5)\cdot 2 \cdot \sqrt{9-y^2}\,(13-y)\,dy$

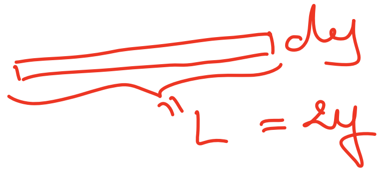
$\Rightarrow F = 125\left[13\underbrace{\int_{-3}^{3}\sqrt{9-y^2}\,dy}_{\frac{1}{2}\pi\cdot r^2} - \underbrace{\int_{-3}^{3} y\sqrt{9-y^2}\,dy}_{0}\right]$

$= \boxed{125 \cdot 13 \cdot \frac{1}{2}\pi \cdot 3^2} = \cdots = \#\,lbs.$

Ex5: A flat isosceles right triangular plate with base 6 ft and height 3 ft is submerged vertically, base up, 2 ft below the surface of a swimming pool. Find the force exerted by the water against one side of the plate.







where $\frac{L}{6} = \frac{y}{3} \Rightarrow L = 2y.$

$$A = 2y\,dy$$

$$d = 5 - y$$

$$V = Ad = 2y(5-y)\,dy$$

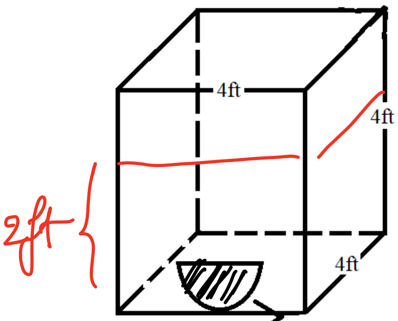
$$F = \int_0^3 (62.5)(2y)(5-y)\,dy = 125\int_0^3 (5y - y^2)\,dy$$

$$= 125 \cdot \left[\frac{5}{2}(3)^2 - \frac{1}{3}(3)^3\right]$$

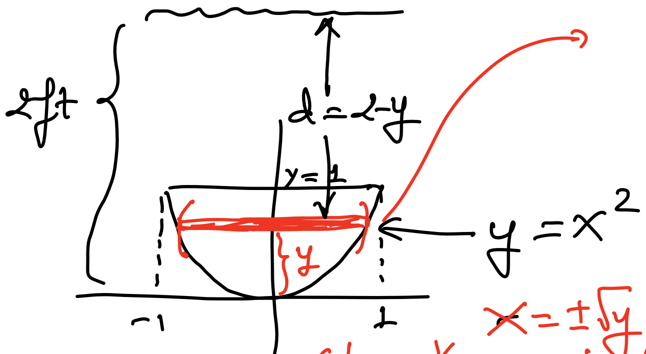
$$= \ldots = \#\text{ lbs.}$$

Ex6: The cubical metal tank shown here has a parabolic gate, held in place by bolts and designed to withstand a fluid force of 160 lb without rupturing. The liquid you plan to store has a weight density of 50 lb/ft^3.

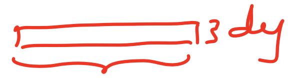
a) What is the fluid force on the gate when the liquid is 2 ft deep?



$y = x^2 \ for \ -1 \le x \le 1$



$x = \pm\sqrt{y}$



$L = \sqrt{y} - (-\sqrt{y})$
$= 2\sqrt{y}.$

$A = 2\sqrt{y}\,dy$

$d = 2-y$

$V = 2\sqrt{y}\cdot(2-y)dy$

$F = 50\cdot 2\int_0^1 \sqrt{y}\,(2-y)dy$

$= 100\int_0^1 (2y^{1/2} - y^{3/2})dy$

$= 100\cdot\left[2\,\frac{2}{3}y^{3/2} - \frac{2}{5}y^{5/2}\right]_0^1$

$= 100\left[\frac{4}{3} - \frac{2}{5}\right] = \underline{93.33\text{ lb}}.$

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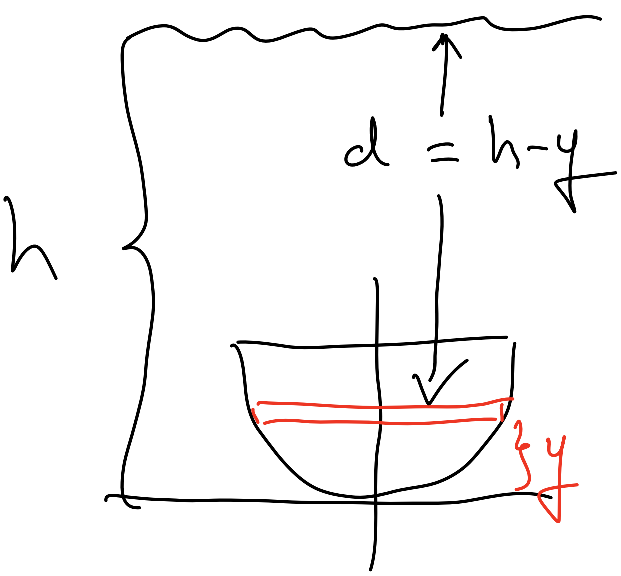
b) What is the maximum height to which the container can be filled without exceeding its design limitation?



$L = 2\sqrt{y}$

$A = 2\sqrt{y}\,dy$

$d = h-y$

$V = 2\sqrt{y}\,(h-y)\,dy.$

$F = 50\cdot 2\int_0^1 \sqrt{y}\,(h-y)dy$

$\Rightarrow F = 100\int_0^1 (hy^{1/2} - y^{3/2})\,dy$

$$= 100\left[h\cdot\frac{2}{3}y^{3/2} - \frac{2}{5}y^{5/2}\right]_0^1$$

$$F = \frac{\cancel{100}\left[\frac{2h}{3} - \frac{2}{5}\right]}{\cancel{100}} = \frac{160}{100}.$$

$$\frac{2h}{3} - \frac{2}{5} = 1.6$$

$$h = \frac{3}{2}\left(1.6 + \frac{2}{5}\right) = 3\ ft.$$