$$f(x) = \frac{2}{x+7} - \frac{5}{x-1} = \frac{2x-2-5x-35}{(x+7)(x-1)} = \int \frac{-3x-37}{(x+7)(x-1)}\,dx$$

Ex: Find a partial fraction deposition.

$$f(x) = \int \frac{2x-1}{x^2-x-20} = \int \frac{2x-1}{(x+4)(x-5)} = \frac{A}{x+4} + \frac{B}{x-5}$$

$x+4=0$, $x=-4$; $x-5=0$, $x=5$

$$A\Big|_{x=-4} = \frac{-8-1}{-4-5} = 1$$

$$B\Big|_{x=5} = \frac{10-1}{5+4} = 1$$

$$\Rightarrow f(x) = \int \frac{1}{x+4} + \int \frac{1}{x-5}.$$

Ex: Evaluate the following:

a) $\int \frac{11x+5}{6x^2+x-2}dx = \int \frac{11x+5}{(2x-1)(3x+2)}dx = \int \left(\frac{A}{2x-1} + \frac{B}{3x+2}\right)dx.$

$$A\Big|_{x=\frac{1}{2}} = \frac{\frac{11}{2}+5}{\frac{3}{2}+2}\cdot\frac{2}{2} = \frac{11+10}{3+4} = 3$$

$$B\Big|_{x=-\frac{2}{3}} = \frac{-\frac{22}{3}+5}{-\frac{4}{3}-1}\cdot\frac{3}{3} = \frac{-22+15}{-4-3} = 1$$

$$\Rightarrow \int\left(\frac{3}{2x-1} + \frac{1}{3x+2}\right)dx = \frac{3}{2}\ln|2x-1| + \frac{1}{3}\ln|3x+2| + C.$$

Note: $\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b| + C$

b) $\int \frac{43x-x^2-30}{x^3-2x^2-15x}dx = \int \frac{43x-x^2-30}{x(x-5)(x+3)}dx = \int\left(\frac{A}{x}+\frac{B}{x-5}+\frac{C}{x+3}\right)dx,$

$A\Big|_{x=0} = \frac{-30}{(-5)(3)} = 2.$

$B\Big|_{x=5} = \frac{43(5)-25-30}{5(8)} = 4$

$C\Big|_{x=-3} = \frac{43(-3)-9-30}{-3(-8)} = -7$

$= \int\left(\frac{2}{x}+\frac{4}{x-5}-\frac{7}{x+3}\right)dx$

$= \boxed{2\ln|x|+4\ln|x-5|-7\ln|x+3|+C}$

$= \ln\left|\frac{x^2\cdot(x-5)^4}{(x+3)^7}\right|+C.$

c) $\int \frac{23x^2+x+23}{(x+1)(5x^2+4)}dx$

$\int \frac{1}{1+x^2}dx = \tan^{-1}x+C$

$\int \frac{1}{a^2+x^2}dx = \frac{1}{a^2}\int \frac{1}{1+\left(\frac{x}{a}\right)^2}dx$

let $u=\frac{x}{a} \Rightarrow du=\frac{1}{a}dx$
$a\,du=dx$

$= \frac{1}{a^2}\int \frac{1}{1+u^2}\cdot a\,du = \frac{1}{a}\int\frac{1}{1+u^2}du = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

1. $\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b|+C$

2. $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$

$\int \frac{23x^2+x+23}{(x+1)(5x^2+4)}dx = \int\left(\frac{A}{x+1}+\frac{Bx+C}{5x^2+4}\right)dx.$

$23x^2+x+23 = A(5x^2+4)+(x+1)(Bx+C)$
$= (5A+B)x^2+(C+B)x+4A+C.$

$\begin{cases}5A+B=23\\ C+B=1\\ 4A+C=23\end{cases}$

$A\Big|_{x=-1} = \frac{23-1+23}{5+4} = 5$

$5(5)+B=23 \Rightarrow B=-2.$

$C+(-2)=1 \Rightarrow C=3.$

$= \int\left(\frac{5}{x+1}+\frac{-2x+3}{5x^2+4}\right)dx = \int\frac{5}{x+1}dx - \frac{2}{10}\int\frac{10x}{5x^2+4}dx + 3\int\frac{1}{4+5x^2}dx.$

$= \underbrace{5\ln|x+1| - \frac{1}{5}\ln(5x^2+4)}_{A} + \frac{3}{5}\int\frac{1}{\frac{4}{5}+x^2}dx.$

$= A + \frac{3}{5}\cdot\frac{\sqrt{5}}{2}\cdot\tan^{-1}\left(\frac{\sqrt{5}}{2}x\right)+C.$

Note: $\int \frac{P(x)}{Q(x)}dx$ If $\deg(P(x)) \geq \deg(Q(x)) \Rightarrow$ Perform the long division b/4 partial fraction.

d) $$\int \frac{2x^3 - 4x^2 - x - 3}{x^2 - 2x - 3}dx$$

$$\begin{array}{r} 2x \\ x^2 - 2x - 3 \,\overline{)\, 2x^3 - 4x^2 - x - 3} \\ -\;\; 2x^3 - 4x^2 - 6x \\ \hline 5x - 3 \end{array}$$ ← Remainder.

$$\int \left(2x + \frac{5x-3}{x^2-2x-3}\right)dx = x^2 + \int \frac{5x-3}{(x-3)(x+1)}dx = x^2 + \int \left(\frac{A}{x-3} + \frac{B}{x+1}\right)dx$$

$$A\Big|_{x=3} = \frac{15-3}{4} = 3.$$

$$B\Big|_{x=-1} = \frac{-5-3}{-1-3} = 2.$$

$$= x^2 + \int \left(\frac{3}{x-3} + \frac{2}{x+1}\right)dx$$

$$= x^2 + 3\ln|x-3| + 2\ln|x+1| + C.$$

e) $$\int \frac{x^2 dx}{(x-1)(x^2+2x+1)} = \int \frac{x^2 dx}{(x-1)(x+1)^2} = \int \left[\frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}\right]dx.$$

$$A\Big|_{x=1} = \frac{1}{4}$$

$$C\Big|_{x=-1} = \frac{1}{-2} = -\frac{1}{2}$$

To find B $\Rightarrow$ plug in $x = \# \neq \pm 1$

$x = 0 \Rightarrow 0 = -A + B + C.$

$$0 = -\frac{1}{4} + B - \frac{1}{2} \Rightarrow B = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}.$$

$$\int \left[\frac{1/4}{x-1} + \frac{3/4}{x+1} - \frac{1/2}{(x+1)^2}\right]dx =$$

$$= \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| - \frac{1}{2}\int \frac{1}{(x+1)^2}dx$$

let $u = x+1$
$du = dx$
$$\int \frac{1}{u^2}du = \int u^{-2}du = \frac{u^{-1}}{-1}$$

$$= \frac{1}{4}\ln|x-1| + \frac{3}{4}\ln|x+1| + \frac{1}{2}\cdot\frac{1}{x+1} + C.$$

f) $\int \frac{e^{4t}+2e^{2t}-e^{t}}{e^{2t}+1}\,dt = \int \frac{e^{3t}+2e^{t}-1}{e^{2t}+1}\cdot e^{t}dt$. $\left\{ \begin{array}{l} \text{Let } u = e^{t} \\ du = e^{t}dt \end{array} \right.$

$$= \int \frac{u^{3}+2u-1}{u^{2}+1}\,du.$$

$$\begin{array}{r|l} & u \\ \hline u^2+1 & u^3+0u^2+2u-1 \\ & -u^3 \qquad\quad u \\ \hline & \qquad\qquad u-1 \end{array}$$

$$= \int \left(u + \frac{u-1}{u^{2}+1}\right)du = \int \left(u + \frac{1}{2}\cdot\frac{2u}{u^{2}+1} - \frac{1}{u^{2}+1}\right)du$$

$$= \frac{1}{2}u^{2} + \frac{1}{2}\ln(u^{2}+1) - \tan^{-1}(u) + C.$$

$$= \frac{1}{2}e^{2t} + \frac{1}{2}\ln(e^{2t}+1) - \tan^{-1}(e^{t}) + C.$$

g) $\int \frac{\sin x\,dx}{\cos^{2}x+\cos x-2}$ $\left\{ \begin{array}{l} \text{Let } u = \cos x \\ du = -\sin x\,dx \\ -du = \sin x\,dx. \end{array} \right.$

$$= \int \frac{-du}{u^{2}+u-2} = -\int \frac{1}{(u+2)(u-1)}du = -\int \left(\frac{A}{u+2} + \frac{B}{u-1}\right)du.$$

$$\left. \begin{array}{l} A\big|_{u=-2} = \frac{1}{-3} = -\frac{1}{3} \\ B\big|_{u=1} = \frac{1}{3} \end{array} \right\} = -\int \left(\frac{-1/3}{u+2} + \frac{1/3}{u-1}\right)du.$$

$$= -\left[-\frac{1}{3}\ln|u+2| + \frac{1}{3}\ln|u-1|\right] + C.$$

$$= \underbrace{\frac{1}{3}\Big[\ln|\cos x+2| - \ln|\cos x-1|\Big]}_{\ln\sqrt[3]{\frac{\cos(x)+2}{\cos(x)-1}}} + C.$$

$$\ln\sqrt[3]{\frac{\cos(x)+2}{\cos(x)-1}} + C.$$

Rationalizing Substitutions:

Ex: Evaluate the following:

a) $\int \frac{\sqrt{x+4}}{x}dx$ $\left\{ \begin{array}{l} \text{let } u=\sqrt{x+4} \Rightarrow u^2 = x+4 \\ 2u\,du = dx \;\; ; \;\; x=u^2-4 \end{array} \right.$

$$= \int \frac{u}{u^2-4}\cdot 2u\,du = 2\int \frac{u^2-4+4}{u^2-4}\,du = 2\int\left[1+\frac{4}{(u-2)(u+2)}\right]du$$

$$=2\int\left[1+\frac{A}{u-2}+\frac{B}{u+2}\right]du \qquad \left\{ \begin{array}{l} A\big|_{u=2} = \frac{4}{4}=1 \\ B\big|_{u=-2}=\frac{4}{-4}=-1 \end{array} \right.$$

$$=2\int\left(1+\frac{1}{u-2}-\frac{1}{u+2}\right)du = 2\left[u+\ln|u-2|-\ln|u+2|\right]+C$$

$$=2\left[\sqrt{x+4}+\ln\left|\sqrt{x+4}-2\right|-\ln\left|\sqrt{x+4}+2\right|\right]+C$$

b) $\int\frac{1}{\sqrt[3]{x}+\sqrt[4]{x}}dx$

Try this: let $x=u^{12}$

b) $$\int \frac{1}{4\sqrt[3]{x}+\sqrt{x}}\,dx = \int\frac{1}{4\sqrt[3]{u^6}+\sqrt{u^6}}\cdot 6u^5\,du$$

Let $x=u^6$
$dx=6u^5\,du$

$$=6\int\frac{u^5}{4u^2+u^3}\,du = 6\int\frac{u^3}{u+4}\,du$$

$1u^3+0u^2+0u+0$

Synthetic Division:

	u^3			
-4	1	0	0	0
		-4	16	-64
	1	-4	16	$\boxed{-64}$ remainder

$u^2-4u+16-\frac{64}{u+4}$

$\frac{7}{3}=2+\frac{1}{3}$

$$=6\int\left(u^2-4u+16-\frac{64}{u+4}\right)du$$

$$=6\left[\frac{1}{3}u^3-2u^2+16u-64\ln|u+4|\right]+C$$

let $(x) = (u^6) \Rightarrow u = x^{1/6}$

$$= 6\left[\frac{1}{3}\left(x^{1/6}\right)^3 - 2\left(x^{1/6}\right)^2 + 16x^{1/6} - 64\ln\left|x^{1/6}+4\right|\right] + C$$

$$= 6\left[\frac{1}{3}\sqrt{x} - 2\sqrt[3]{x} + 16\sqrt[6]{x} - 64\ln\left|\sqrt[6]{x}+4\right|\right] + C$$

Quiz # 1 "Group".

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break out - room: $\boxed{\text{4 people}}$

$\boxed{A}$ $\boxed{B \quad C \quad D}$

piece of paper

Ⓐ, B, C, D
Complete Quiz.

Ⓐ B C D

45 – 1 hr

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1. Differentiate $f(x)$

$$\begin{cases} (uv)' = \dots \\ \left(\frac{u}{v}\right)' = \dots \\ (u \circ v)' = \dots \end{cases}$$

(No need to simplify)

a)

b)

2. Find $\frac{dy}{dx}$ implicitly

A B

3. Integrate the following.

a) u-sub

case 1 ← short cut
" 2
" 3.

b) parts

c) Trig. ← u= ; double angle formula.

d) Trig. sub. ← Complete square

e) partial fraction ← Repeated linear factors,

f) others. ← u-sub → - partial fract or - synthetic Div.

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at 4:30pm