Section 7.5 *Strategy for Integration*

1. Integration techniques:
   - a) Substitution
   - b) Parts
   - c) Trig. Integrals
   - d) Trig. Subs
   - e) Partial Fraction
   - f) Others.

Ex: Integrate the following:

a) $\int \sqrt{2+\sqrt{3x+1}}\,dx$

Nested root $\Rightarrow$ $u = \sqrt{2+\sqrt{3x+1}}$

$u^2 = 2+\sqrt{3x+1}$

$(u^2-2)^2 = (\sqrt{3x+1})^2$

$u^4 - 4u^2 + 4 = 3x+1$

$(4u^3 - 8u)\,du = 3dx$.

$\cos(\sqrt[4]{x})$

b) $\int e^{\sqrt[4]{x}}dx \Rightarrow$ $u = \sqrt[4]{x}$

$u^4 = x$

$4u^3 du = dx$

$\int e^u \cdot 4u^3 du = 4\int e^u \cdot u^3 du \leftarrow$ parts.

u^3	$(+)\ e^u$
$3u^2$	$(-)\ e^u$
$6u$	$(+)\ e^u$
6	$(-)\ e^u$
0	e^u

c) $\int \frac{\sin(3x)}{\cos^2(3x)+4\cos(3x)-21}dx$

$$\left\{ \begin{array}{l} \text{let } u = \cos(3x) \\ du = -3\sin(3x)\,dx \\ -\frac{1}{3}du = \sin(3x)\,dx \end{array} \right.$$

$$= -\frac{1}{3}\int \frac{du}{u^2+4u-21} = -\frac{1}{3}\int \frac{1}{(u+7)(u-3)}du$$

$$\rightarrow = -\frac{1}{3}\int \left(\frac{A}{u+7} + \frac{B}{u-3}\right)du.$$

d) $\int \frac{2x-3}{\left[4x^2+4x-15\right]^{3/2}}dx$

$$\underline{4x^2}+4x-15 = \underbrace{(2x)^2+2(2x)+1}-16$$

$$= (2x+1)^2-4^2 \rightarrow \text{let } \underline{2x+1 = 4\sec\theta}$$

① Others:

multiply top & bot by the conjugate of the denominator.

e) $\int \frac{dx}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x} =$

$$= \int \frac{1+\cos x}{1-\cos^2 x}\,dx = \int \frac{1+\cos x}{\sin^2 x}\,dx = \int \left[\csc^2 x + \cot x \csc x\right] dx.$$

$$= -\cot x - \csc x + C.$$

f) $\int \frac{dx}{\sqrt{x}+4\sqrt[3]{x}}$ > let $x = u^6$

g) $\int \sin^{-1}(5x)\,dx = x\sin^{-1}(5x) - \int \frac{5x}{\sqrt{1-25x^2}}\,dx$ $\left\{ \begin{array}{l} u = \sqrt{1-25x^2} \\ \ldots \end{array} \right.$

$\sin^{-1}(5x)$	dx
$\frac{5}{\sqrt{1-25x^2}}\,dx$	x

h) $\int \frac{2x^3 - 2x^2 - 25x - 31}{x^2 - x - 12} dx = \int \left[2x - \frac{x+31}{(x+3)(x-4)} \right] dx.$

partial fraction,

$$\begin{array}{r} 2x \\ 1 \cdot x^2 - x - 12 \overline{\big) 2x^3 - 2x^2 - 25x - 31} \\ -(2x^3 - 2x^2 - 24x) \\ \hline -x - 31 \end{array}$$

i) $\int \ln(4x+5)\, dx = \underbrace{x \ln(4x+5)}_{A} - \int \frac{\boxed{4x+5}-5}{\boxed{4x+5}} dx$

$\ln(4x+5)$	dx
$\frac{4}{4x+5} dx$	x

$-\int$

$= A - \int \left(1 - \frac{5}{4x+5}\right) dx$

$= A - \left[x - \frac{5}{4} \ln|4x+5|\right] + C,$

j) $\int (7x^3 - 5x + 3) \sin(2x)\, dx$

$7x^3 - 5x + 3$	(+)	$\sin(2x)\,dx$
$21x^2 - 5$	(−)	$-\frac{1}{2}\cos(2x)$
$42x$	(+)	$-\frac{1}{4}\sin(2x)$
42	(−)	$\frac{1}{8}\cos(2x)$
0		$\frac{1}{16}\sin(2x)$.

...

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others.

k) $\int\sqrt{\frac{1-x}{1+x}}dx = \int \frac{\sqrt{1-x}}{\sqrt{1+x}} \cdot \frac{\sqrt{1-x}}{\sqrt{1-x}} dx = \int \frac{1-x}{\sqrt{1-x^2}} dx$

$$= \underbrace{\int \frac{1}{\sqrt{1-x^2}} dx}_{} - \int \frac{x}{\sqrt{1-x^2}} dx$$

let $u = \sqrt{1-x^2}$
$u^2 = 1 - x^2$
$2udu = -2xdx$
$-udu = xdx.$

$$= \sin^{-1}x + \int \frac{\cancel{u}du}{\cancel{u}}.$$

$$= \sin^{-1}x + \sqrt{1-x^2} + C.$$

l) $\int x^8 \sqrt[4]{2x^3+1}\,dx$ → let $u = \sqrt[4]{2x^3+1}$.

$\int x^6 \cdot \sqrt[4]{2x^3+1} \cdot x^2 dx$

$u^4 = 2x^3 + 1 \Rightarrow x^3 = \frac{1}{2}(u^4-1)$

$4u^3du = 6x^2dx.$ $\quad x^6 = \frac{1}{4}(u^4-1)^2.$

$\frac{2}{3}u^3du = x^2dx.$

$$= \int \frac{1}{4}(u^4-1)^2 \cdot u \cdot \frac{2}{3}u^3 du \ldots$$

m) $\int \sqrt{1+\cos(5x)}\, dx.$

$$\int \sqrt{1+\cos(5x)} \cdot \frac{\sqrt{1-\cos(5x)}}{\sqrt{1-\cos(5x)}} dx$$

$$\int \frac{\sqrt{1-\cos^2(5x)}}{\sqrt{1-\cos(5x)}}\,dx = \int \frac{\sin(5x)}{\sqrt{1-\cos(5x)}}\,dx$$

let $u = \sqrt{1-\cos(5x)}$

$$u^2 = 1-\cos(5x)$$

$$2u\,du = 5\sin(5x)\,dx$$

$$\frac{2}{5}u\,du = \sin(5x)\,dx$$

$$= \frac{2}{5}\int \frac{\cancel{u}\,du}{\cancel{u}} = \frac{2}{3}\sqrt{1-\cos(5x)} + C$$

#Q4#

Half-Angle formula.

$$\cos^2\theta = \frac{1+\cos(2\theta)}{2} \;;\; \sin^2\theta = \frac{1-\cos(2\theta)}{2}$$

$$\theta = \frac{x}{2}$$

$$\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos(x)}{2} \quad ; \quad \sin^2\left(\frac{x}{2}\right) = \frac{1-\cos(x)}{2}$$

$$\boxed{\cos\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1+\cos(x)}{2}} \quad ; \quad \sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos(x)}{2}}}$$

$$\int \sqrt{1+\cos(5x)}\,dx$$

$$= \int \sqrt{\frac{1+\cos(5x)}{2} \cdot 2}\,dx.$$

$$= \sqrt{2} \int \sqrt{\frac{1+\cos(5x)}{2}}\,dx.$$

$$= \sqrt{2} \int \cos\left(\frac{5x}{2}\right) dx$$

$$= \sqrt{2} \cdot \frac{2}{5} \cdot \sin\left(\frac{5x}{2}\right) + C.$$