Math 180  Maple Project

Email your project to ftran@mtsac.edu with your full name and class time on the subject line. Not hard copies are accepted. Do not print out your project.

1. **Define functions:**
   \[ f := x \rightarrow \sin(2x - 3) \cdot \exp(x - 1); \]
   \[ x \rightarrow \sin(2x - 3) \cdot e^{x - 1} \]

   **Evaluate function:**
   \[ f\left( \frac{\pi}{2} \right); \]
   \[ \sin(3) \cdot e^{\frac{1}{2}} \cdot \pi - 1 \]

   **Sketch the graph:**
   \[ \text{plot}(f(x), x = -\pi..\pi, y = -10..10) \]

**Taking limit:**

a) Define \( f(x) = e^{2-x} \tan^{-1}(3x + 5) \) and \( f(x) = \frac{2x^2 + 7x - 15}{3x^2 + 16x + 5} \) then evaluate \( \lim_{x \rightarrow \infty} f(x) \) and \( \lim_{x \rightarrow -5} g(x) \)

> \[ f := x \rightarrow \exp(2 - x) \arctan(x); \]
   \[ x \rightarrow e^{2 - x} \arctan(x) \]

\[ \text{limit}(f(x), x = \infty); \]
   \[ 0 \]

\[ g := x \rightarrow \frac{(2x^2 + 7x - 15)}{3x^2 + 16x + 5}; \]
   \[ x \rightarrow \frac{2x^2 + 7x - 15}{3x^2 + 16x + 5} \]

\[ \text{limit}(g(x), x = -5); \]
1. The rules of exponents tell us that \( a^0 = 1 \) if \( a \) is any number different from zero. They also tell us that \( 0^n = 0 \) if \( n \) is any positive number. If we tried to extend these rules to include the case \( 0^0 \), we would get conflicting results. The first rule would say \( 100 = 1 \), whereas the second would say \( 0^0 = 0 \). We are not dealing with a question of right or wrong here. Neither rule applies as it stands, so there is no contradiction. We could, in fact, define \( 0^0 \) to have any value we wanted as long as we could persuade others to agree.

For the following functions, use the above steps, define the functions, sketch their graphs over the indicated interval, and then evaluate the limit.

a) \( f(x) = x^2 \), \([-1,2]\) \( \lim_{x \to 0^-} f(x) \)

b) \( f(x) = \frac{3x + \pi}{\sin(x + \frac{\pi}{3})} \), \([-\pi, \pi]\) \( \lim_{x \to \frac{\pi}{3}^-} f(x) \)

c) \( f(x) = \frac{\ln(\csc x)}{(x - \frac{\pi}{2})^2} \), \([-\pi, \pi]\) \( \lim_{x \to \frac{\pi}{3}^-} f(x) \)

d) \( f(x) = \frac{3^x - 1}{2^x - 1} \), \([-1,2]\) \( \lim_{x \to 0^-} f(x) \)

e) \( f(x) = \left( \frac{3x + 1}{7x + 5} \right)^{\frac{1}{x}} \), \([0,3]\) \( \lim_{x \to 0^-} f(x) \)

f) \( f(x) = (e^x + x)^{\frac{1}{x}} \), \([0,3]\) \( \lim_{x \to 0^-} f(x) \)

g) \( f(x) = (\ln x)^{\frac{1}{x-e}} \), \([0,5]\) \( \lim_{x \to e^-} f(x) \)

h) \( f(x) = \frac{\sec x}{\tan x} \), \([0,\pi]\) \( \lim_{x \to \frac{\pi}{2}^-} f(x) \)

2. Evaluate the following limits:

a) \( \lim_{x \to \infty} (1 + 2x)^{\frac{1}{2 \ln x}} \)

b) \( \lim_{x \to \infty} \frac{2^x - 3^x}{3^x + 4^x} \)

c) \( \lim_{x \to \infty} \frac{x - \sqrt{x^2 + x}}{x} \)

d) \( \lim_{x \to \infty} \left( \frac{5x - 7}{2x + 3} \right)^x \)

e) \( \lim_{x \to \infty} \left( \frac{x^2 + 7}{x + 2} \right)^{\frac{1}{x}} \)

f) \( \lim_{x \to 0^-} e^{-x^{1/x}} \)

3. Differentiation:

Define a function and then differentiate the function:

\( f(x) = x^3 \cos(5x^2 - 3) \) and then find derivative \( \frac{df}{dx} \), \( \frac{d^2 f}{dx^2} \), \( \frac{d^3 f}{dx^3} \), \( \frac{d^4 f}{dx^4} \);

\( f := x \rightarrow x^3 \cos(5x^2 - 3); \)

\( x \rightarrow x^3 \cos(5x^2 - 3) \)

\( fp := \text{diff}(f(x), x); \)

\( 3x^2 \cos(5x^2 - 3) - 10x^4 \sin(5x^2 - 3) \)

\( fp2 := \text{diff}(f(x), x, x); \)

\( 6x \cos(5x^2 - 3) - 70x^3 \sin(5x^2 - 3) - 100x^5 \cos(5x^2 - 3) \)

\( fp3 := \text{diff}(f(x), x^4); \)

\( -600 \sin(5x^2 - 3) x - 7500x^3 \cos(5x^2 - 3) + 18000x^5 \sin(5x^2 - 3) + 10000x^7 \cos(5x^2 - 3) \)
Do problems: 37 – 42 on page 299.

4. Implicit differentiation:

Ex: \( e^{x^2 y^3 + x - y^2} + \sin(x^3 + y) = 1 \)

> \( a := \exp(x^2 y + x - y^2) + \sin(x^3 + y) = 1; \)

\[
\frac{e^{2y^2 - y^2 + x} + \sin(x^3 + y) = 1}{\text{implicitdiff}(a, y, x);}
\]

Determine \( \frac{dy}{dx} \) implicitly of the following:

a) \( \sqrt{2x^3 y^2 - x - 5y - \tan^{-1}(x^3 y)} = 2 \)

b) \( e^{\sin(x^3 y)} = \sin^2(x^3 + 5y^2) + x - y \)

5. A cone of height \( h \) and radius \( r \) is constructed from a flat circular disk of radius \( a \) in. by removing a section \( AOC \) of arch length \( x \) in. and then connecting the edges \( OA \) and \( OC \).

a) Find a formula for the volume \( V \) of the cone in terms of \( x \) and \( a \).

b) Find radius \( r \) and height \( h \) in the cone of maximum volume for \( a = 4, 5 \) and 6.

c) How should we cut the disk of radius 4in (\( a = 4 \)in.) so that the volume of the cone maximum?

6. Find area under a function by definition:

\[
\int_{a}^{b} f(x)dx \approx \lim_{n \to \infty} \left( \sum_{i=1}^{n} f(x_i) \Delta x \right)
\]

Ex: Evaluate the integral \( \int_{-1}^{3} (x^3 - 2x + 5) \sin(2x + 1) dx \) by using limit definition:

> \( f := x \rightarrow (x^3 - 2x + 5) \cdot \sin(2x + 1); \)

\( a := -1; b := 3; \)

\( \Delta x := \frac{(b - a)}{n}; \)
6. Now, using the procedure outline about to evaluate the following integrals by using the limit definition:

a) \[ \int_{-\pi/2}^{0} (2x + 135\sin x) \, dx \]

b) \[ \int_{-7}^{5} (2x + 15\ln x) \, dx \]

c) \[ \int_{3}^{1} 3e^{-x} \, dx \]

d) \[ \int_{-\pi/2}^{0} 12\cos^3 x \, dx \]

7. Antiderivative:

Ex: Evaluate: \[ \int_{0}^{\pi} (x^2 + 1)\sin^2 (x + 1) \, dx \]

> \( f := x \rightarrow (x^2 + 1)\cdot\sin^2(x + 1); \)

\( x \rightarrow (x^2 + 1) \sin(x + 1)^2 \)

\( a := \text{int}(f(x), x = 0..\pi); \)

\( -\frac{1}{2} + \frac{1}{2} \cos(1)^2 + \frac{1}{2} \sin(1)^2 - \frac{1}{2} \cos(1) \sin(1) \pi^2 - \frac{1}{2} \cos(1)^2 \pi + \frac{1}{6} \pi^3 + \frac{3}{4} \pi \)

> \( \text{evalf}(a); \)

Use the above technique do problems: 59 – 69 on page 414