Quiz #1 ***Math 181*** ***Name:***

6/24/21

Show all your work clearly. No Work, No Credit.

1. Differentiate the following functions. (No need to simplify)

a) $f(x)=\tan^4\left(\dfrac{\sqrt{7x^5+3x^2-4}}{\sin^{-1}(2x-3)}\right)$ (4pts)

$$f'(x)=4\tan^3\left(\frac{\sqrt{7x^5+3x^2-4}}{\sin^{-1}(2x-3)}\right)\sec^2\left(\frac{\sqrt{7x^5+3x^2-4}}{\sin^{-1}(2x-3)}\right)\cdot A$$

$$A=\frac{\frac{1}{2}\left(7x^5+3x^2-4\right)^{-1/2}\left(35x^4+6x\right)\sin^{-1}(2x-3)-\frac{2}{\sqrt{1-(2x-3)^2}}\cdot\sqrt{7x^5+3x^2-4}}{\left(\sin^{-1}(2x-3)\right)^2}$$

b) $f(x)=4^{\sin^{-1}(5x^4+1)}\left(\sec(3x)+x^2+1\right)$ (4 pts)

$$f'(x)=4^{\sin^{-1}(5x^4+1)}\left[\frac{20x^3}{\sqrt{1-(5x^4+1)^2}}\left(\sec(3x)+x^2+1\right)\ln 4+\sec(3x)\tan(3x)+2x\right]$$

2. Integrate the following: (42pts)

a) $\displaystyle\int_0^{\frac{\sqrt{3}}{3}}\frac{\left(\tan^{-1}(3x)+1\right)^2}{1+9x^2}dx$;

Let $u=\tan^{-1}(3x)+1$

$$\begin{cases} x=\frac{\sqrt{3}}{3}\Rightarrow u=\tan^{-1}(\sqrt{3})+1=\frac{\pi}{3}+1 \\ x=0\Rightarrow u=\tan^{-1}(0)+1=1 \end{cases}$$

$$du=\frac{3}{1+9x^2}dx \Rightarrow \frac{du}{3}=\frac{1}{1+9x^2}dx$$

$$=\int_1^{\frac{\pi}{3}+1}u^2\cdot\frac{du}{3}=\frac{1}{3}\int_1^{\frac{\pi}{3}+1}u^2du=\frac{1}{3}\cdot\frac{1}{3}u^3\Big|_1^{\frac{\pi}{3}+1}$$

$$=\frac{1}{9}\left[\left(\frac{\pi}{3}+1\right)^3-1\right]=\cdots=\#$$

b) $\int x^9 \sqrt[5]{2x^5+3}\,dx$ $\left\{\begin{array}{l} u = \sqrt[5]{2x^5+3} \Rightarrow u^5 = 2x^5+3 \longrightarrow x^5 = \frac{1}{2}(u^5-3) \\ 5u^4du = 10x^4dx \Rightarrow \frac{1}{2}u^4du = x^4dx \end{array}\right.$

$$= \int x^5 \cdot \sqrt[5]{2x^5+3} \cdot x^4dx = \int \frac{1}{2}(u^5-3)\cdot u \cdot \frac{1}{2}\cdot u^4du.$$

$$= \frac{1}{4}\int (u^{10} - 3u^5)\,du = \frac{1}{4}\left[\frac{1}{11}u^{11} - \frac{1}{2}u^6\right] + C$$

$$= \frac{1}{4}\left[\frac{1}{11}\left(\sqrt[5]{2x^5+3}\right)^{11} - \frac{1}{2}\left(\sqrt[5]{2x^5+3}\right)^6\right] + C$$

c) $\int e^{-3x}\left(4x^3-5x+7\right)dx$

$4x^3-5x+7$	$(+)\ e^{-3x}dx$
$12x^2-5$	$(-)\ -\frac{1}{3}e^{-3x}$
$24x$	$(+)\ \frac{1}{9}e^{-3x}$
24	$(-)\ -\frac{1}{27}e^{-3x}$
0	$\frac{1}{81}e^{-3x}$

$$= e^{-3x}\left[-\frac{1}{3}(4x^3-5x+7) - \frac{1}{9}(12x^2-5) - \frac{1}{27}(24x) - \frac{1}{81}(24)\right] + C$$

d) $\int \ln(5x+4)\,dx = x\ln(5x+4) - \int \frac{5x+4-4}{5x+4}dx$

$\ln(5x+4)$	dx
$\frac{5}{5x+4}dx$	x

$$= x\ln(5x+4) - \int\left(1 - \frac{4}{5x+4}\right)dx$$

$$= x\ln(5x+4) - \left(x - \frac{4}{5}\ln|5x+4|\right) + C$$

e) $\displaystyle\int \frac{\sin^3(2x)}{\sqrt[5]{\cos^3(2x)}}dx = \int \frac{1-\cos^2(2x)}{\sqrt[5]{\cos^3(2x)}}\sin(2x)dx = \int \frac{1-u^2}{u^{3/5}}\cdot\frac{du}{-2}$

Let $u = \cos(2x)$

$du = -2\sin(2x)dx$

$\frac{du}{-2} = \sin(2x)dx$

$$= -\frac{1}{2}\int\left(u^{-3/5} - u^{7/5}\right)du$$

$$= -\frac{1}{2}\left[\frac{5}{2}u^{2/5} - \frac{5}{12}u^{12/5}\right] + C$$

$$= -\frac{1}{2}\left[\frac{5}{2}\left(\cos(2x)\right)^{2/5} - \frac{5}{12}\left(\cos(2x)\right)^{12/5}\right] + C$$

f) $\displaystyle\int \cos\left(\sqrt[4]{x}\right)dx$; Let $u = \sqrt[4]{x} \Rightarrow u^4 = x \Rightarrow 4u^3du = dx$

$$= \int \cos(u)\cdot 4u^3du = 4\int u^3\cos(u)du$$

u^3	$(+)$	$\cos(u)du$
$3u^2$	$(-)$	$\sin(u)$
$6u$	$(+)$	$-\cos(u)$
6	$(-)$	$-\sin(u)$
0		$\cos(u)$

$$= 4\left[u^3\sin(u) + 3u^2\cos(u) - 6u\sin(u) - 6\cos(u)\right] + C$$

$$= 4\left[\left(\sqrt[4]{x}\right)^3\sin\left(\sqrt[4]{x}\right) + 3\left(\sqrt[4]{x}\right)^2\cos\left(\sqrt[4]{x}\right) - 6\sqrt[4]{x}\sin\left(\sqrt[4]{x}\right) - 6\cos\left(\sqrt[4]{x}\right)\right] + C$$

g) $\displaystyle\int \frac{7x^3-6x^2-19x-8}{x^2-2x-3}dx = \int\left(7x+8+\frac{18x+16}{x^2-2x-3}\right)dx = \int\left(7x+8+\frac{18x+16}{(x-3)(x+1)}\right)dx$

$$\begin{array}{r|l} & 7x+8 \\ \hline x^2-2x-3 & 7x^3-6x^2-19x-8 \\ & -(7x^3-14x^2-21x) \\ \hline & 8x^2+2x-8 \\ & -(8x^2-16x-24) \\ \hline & 18x+16 \end{array}$$

$$= \int\left(7x+8+\frac{A}{x-3}+\frac{B}{x+1}\right)dx$$

$A\big|_{x=3} = \frac{48+16}{4} = 16$; $B\big|_{x=-1} = \frac{-18+16}{-4} = \frac{1}{2}$

$$= \int\left(7x+8+\frac{16}{x-3}+\frac{1/2}{x+1}\right)dx$$

$$= \frac{7}{2}x^2 + 8x + 16\ln|x-3| + \frac{1}{2}\ln|x+1| + C$$

h) $\displaystyle\int e^{4x}\sin(5x)\,dx = \underbrace{-\frac{1}{5}e^{4x}\cos(5x)}_{A} + \frac{4}{5}\int e^{4x}\cos(5x)\,dx$

e^{4x}	$\sin(5x)\,dx$
$4e^{4x}\,dx$	$-\frac{1}{5}\cos(5x)$

$$= A + \frac{4}{5}\left[\frac{1}{5}e^{4x}\sin(5x) - \frac{4}{5}\int e^{4x}\sin(5x)\,dx\right]$$

e^{4x}	$\cos(5x)\,dx$
$4e^{4x}\,dx$	$\frac{1}{5}\sin(5x)$

$$= A + \frac{4}{25}e^{4x}\sin(5x) - \frac{16}{25}\int e^{4x}\sin(5x)\,dx$$

$$\Rightarrow \left(1+\frac{16}{25}\right)\int e^{4x}\sin(5x)\,dx = A + \frac{4}{25}e^{4x}\sin(5x)$$

Ans: $\displaystyle\int e^{4x}\sin(5x)\,dx = \frac{25}{41}\left[A + \frac{4}{25}e^{4x}\sin(5x)\right] + C$

i) $\displaystyle\int \frac{14x^2-13x+31}{(x-5)(3x^2+4)}\,dx = \int\left(\frac{A}{x-5} + \frac{Bx+C}{3x^2+4}\right)dx$

$$14x^2 - 13x + 31 = A(3x^2+4) + (x-5)(Bx+C)$$
$$= (3A+B)x^2 + (C-5B)x + 4A - 5C$$

$3A + B = 14$

$C - 5B = -13$

$4A - 5C = 31$

$A\big|_{x=5} = \dfrac{350-65+31}{75+4} = 4$

$16 - 5C = 31 \Rightarrow C = \dfrac{16-31}{5} = -3$

$-3 - 5B = -13 \Rightarrow B = 2$

$$\Rightarrow \int\left(\frac{4}{x-5} + \frac{2x-3}{3x^2+4}\right)dx$$

$$= \int \frac{4}{x-5}\,dx + \frac{2}{6}\int \frac{6x}{3x^2+4}\,dx - \frac{3}{3}\int \frac{1}{\frac{4}{3}+x^2}\,dx$$

$$= 4\ln|x-5| + \frac{1}{3}\ln(3x^2+4) - \frac{\sqrt{3}}{2}\tan^{-1}\left(\frac{\sqrt{3}}{2}x\right) + C$$

j) $\displaystyle\int \frac{3x+7}{\sqrt{9x^2-12x+20}}\,dx = \int \frac{3x-2+9}{\sqrt{(3x-2)^2+16}}\,dx$

$9x^2-12x+20 = (3x)^2 - 2(3x)\cdot 2 + 4 + 16$

$= (3x-2)^2 + 16$

let $3x-2 = 4\tan\theta$

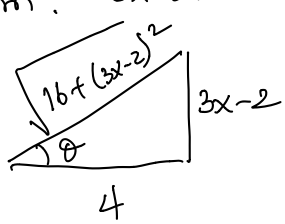
$3dx = 4\sec^2\theta\, d\theta \Rightarrow dx = \frac{4}{3}\sec^2\theta\, d\theta$

$$= \int \frac{4\tan\theta + 9}{\sqrt{16\tan^2\theta + 16}} \frac{4}{3}\sec^2\theta\, d\theta = \frac{4}{3}\int \frac{4\tan\theta + 9}{\sqrt{16\sec^2\theta}} \sec^2\theta\, d\theta = \frac{4}{3}\int \frac{4\tan\theta + 9}{4\sec\theta}\cdot \sec^2\theta\, d\theta$$

$$= \frac{1}{3}\int \left(4\tan\theta\sec\theta + 9\sec\theta\right) d\theta$$

$$= \frac{1}{3}\left[4\sec\theta + 9\ln|\sec\theta + \tan\theta|\right] + C$$

from: $3x-2 = 4\tan\theta \Rightarrow \tan\theta = \frac{3x-2}{4} = \frac{\text{opp}}{\text{Adj.}}$



$\Rightarrow$ Ans: $= \frac{1}{3}\left[4\cdot\frac{\sqrt{16+(3x-2)^2}}{4} + 9\ln\left|\frac{\sqrt{16+(3x-2)^2}}{4} + \frac{3x-2}{4}\right|\right] + C$