Quiz #1 **Math 181** **6/24/21** **Name:** KEY

Show all your work clearly. No Work, No Credit.

1. Differentiate the following functions. (No need to simplify)

a) $f(x)=\sin^3\left(\dfrac{\sqrt{3x^2+5x-4}}{\tan^{-1}(x^3+2)}\right)$ (5 pts)

$$f'(x)=3\sin^2\left(\frac{\sqrt{3x^2+5x-4}}{\tan^{-1}(x^3+2)}\right)\cos\left(\frac{\sqrt{3x^2+5x-4}}{\tan^{-1}(x^3+2)}\right)\cdot A$$

$$A=\frac{\frac{1}{2}(3x^2+5x-4)^{-1/2}(6x+5)\tan^{-1}(x^3+2)-\frac{3x^2}{1+(x^3+2)^2}\cdot\sqrt{3x^2+5x-4}}{\left(\tan^{-1}(x^3+2)\right)^2}$$

b) $f(x)=e^{\sin^{-1}(x^3+1)}(3x^3-4x^2+7)$ (5 pts)

$$f'(x)=e^{\sin^{-1}(x^3+1)}\cdot\left[\frac{3x^2}{\sqrt{1-(x^3+1)^2}}(3x^3-4x^2+7)+9x^2-8x\right]$$

2. Integrate the following: (40 pts = 4pts / each)

a) $\displaystyle\int_0^3\frac{x-2}{\sqrt[3]{3x-1}}\,dx$

$$\begin{cases}\text{let } u=\sqrt[3]{3x-1}\Rightarrow\\ u^3=3x-1\\ 3u^2du=3dx\Rightarrow dx=u^2du\end{cases}\qquad\begin{cases}x=3\Rightarrow u=\sqrt[3]{8}=2\\ x=0\Rightarrow u=\sqrt[3]{-1}=-1\end{cases}$$

$$\int_{-1}^{2}\frac{\frac{1}{3}(u^3+1)-2}{u}\cdot u^2du=\int_{-1}^{2}\left(\frac{1}{3}u^3+\frac{1}{3}-2\right)u\,du$$

$$=\int_{-1}^{2}\left(\frac{1}{3}u^4-\frac{5}{3}u\right)du=\frac{1}{15}u^5-\frac{5}{6}u^2\Big|_{-1}^{2}$$

$$=\left[\frac{32}{15}-\frac{5}{3}+\frac{1}{15}+\frac{5}{6}\right]=\cdots=\#$$

b) $\int x^5 \sqrt[3]{4x^3+1}\,dx$
$$\left\{\begin{array}{l} u = \sqrt[3]{4x^3+1} \\ u^3 = 4x^3+1 \rightarrow x^3 = \frac{1}{4}(u^3-1) \\ 3u^2du = 12x^2dx. \\ \frac{1}{4}u^2du = x^2dx. \end{array}\right.$$

$$= \int x^3 \cdot \sqrt[3]{4x^3+1} \cdot x^2dx = \int \frac{1}{4}(u^3-1)\cdot u \cdot \frac{1}{2}u^2du. = \frac{1}{8}\int (u^6-u^3)\,du.$$

$$= \frac{1}{8}\left[\frac{1}{7}u^7 - \frac{1}{4}u^4\right] + C.$$

$$= \frac{1}{8}\left[\frac{1}{7}\left(\sqrt[3]{4x^3+1}\right)^7 - \frac{1}{4}\left(\sqrt[3]{4x^3+1}\right)^4\right] + C.$$

c) $\int e^{2x}\left(7x^3+2x-3\right)dx$ $= e^{2x}\left[\frac{1}{2}\left(7x^3+2x-3\right) - \frac{1}{4}\left(21x^2+2\right) + \frac{1}{8}\left(42x\right) - \frac{1}{16}(42)\right] + C$

$7x^3+2x-3$	$e^{2x}dx$
$21x^2+2$	$(+)\ \frac{1}{2}e^{2x}$
$42x$	$(-)\ \frac{1}{4}e^{2x}$
42	$(+)\ \frac{1}{8}e^{2x}$
0	$(-)\ \frac{1}{16}e^{2x}$

d) $\int \sin^{-1}(3x)\,dx = x\sin^{-1}(3x) - \int \frac{3x}{\sqrt{1-9x^2}}dx$
$$\left\{\begin{array}{l} u = \sqrt{1-9x^2} \\ u^2 = 1-9x^2 \\ 2udu = -18x\,dx \\ -\frac{1}{3}u\,du = 3x\,dx \end{array}\right.$$

$\sin^{-1}(3x)$	dx
$\frac{3}{\sqrt{1-9x^2}}dx$	x

$$= x\sin^{-1}(3x) + \frac{1}{3}\int \frac{\cancel{u}\,du}{\cancel{u}}. = x\sin^{-1}(3x) + \frac{1}{3}u + C$$

$$= x\sin^{-1}(3x) + \frac{1}{3}\sqrt{1-9x^2} + C$$

e) $\int \sqrt[3]{\cos^4(2x)}\sin^5(2x)\,dx = \int \sqrt[3]{\cos^4(2x)}\cdot\left(1-\cos^2(2x)\right)^2\cdot\sin(2x)\,dx.$

Let $u=\cos(2x)$

$du = -2\sin(2x)\,dx$

$-\frac{du}{2} = \sin(2x)\,dx$

$$= \int u^{4/3}\left(1-u^2\right)^2\cdot\frac{du}{-2}$$

$$= -\frac{1}{2}\int u^{4/3}\left(1-2u^2+u^4\right)du = -\frac{1}{2}\int\left[u^{4/3}-2u^{10/3}+u^{16/3}\right]du$$

$$= -\frac{1}{2}\left[\frac{3}{7}\left(\cos(2x)\right)^{7/3} - 2\cdot\frac{3}{13}\left(\cos(2x)\right)^{13/3} + \frac{3}{19}\left(\cos(2x)\right)^{19/3}\right]+C$$

f) $\int \cos\left(\sqrt[4]{x}\right)dx$ Let $u=\sqrt[4]{x} \Rightarrow u^4 = x \Rightarrow 4u^3\,du = dx$

$$\int \cos u\cdot 4u^3\,du = 4\int u^3\cos u\,du.$$

u^3	$(+)$	$\cos u\,du$
$3u^2$	$(-)$	$\sin u$
$6u$	$(+)$	$-\cos u$
6	$(-)$	$-\sin u$
0		$\cos u$

$$= 4\left[\left(\sqrt[4]{x}\right)^3\sin\left(\sqrt[4]{x}\right) + 3\left(\sqrt[4]{x}\right)^2\cos\left(\sqrt[4]{x}\right) - 6\left(\sqrt[4]{x}\right)\sin\left(\sqrt[4]{x}\right) - 6\cos\left(\sqrt[4]{x}\right)\right] + C$$

g) $\int \frac{3x^3-2x^2-8x+6}{x^2-x-2}dx = \int\left(3x+1+\frac{8-x}{x^2-x-2}\right)dx = \frac{3}{2}x^2+x+\int\frac{8-x}{(x-2)(x+1)}dx$

$$\begin{array}{r|l} & 3x + 1 \\ x^2-x-2 & 3x^3 - 2x^2 - 8x + 6 \\ & -(3x^3 - 3x^2 - 6x) \\ \hline & x^2 - 2x + 6 \\ & -(x^2 - x - 2) \\ \hline & -x + 8 \end{array}$$

$= \frac{3}{2}x^2+x+\int\left(\frac{A}{x-2}+\frac{B}{x+1}\right)dx$

$A\big|_{x=2} = \frac{6}{3} = 2$; $B\big|_{x=-1} = \frac{9}{-3} = -3.$

$$= \frac{3}{2}x^2+x+\int\left(\frac{2}{x-2}-\frac{3}{x+1}\right)dx = \frac{3}{2}x^2+x+2\ln|x-2|-3\ln|x+1|+C$$

h) $\int e^{-2x}\cos(3x)\,dx = \underbrace{\frac{1}{3}e^{-2x}\sin(3x)}_{A} + \frac{2}{3}\int e^{-2x}\sin(3x)\,dx$

$$\begin{array}{c|c} e^{-2x} & \cos(3x)\,dx \\ \hline -2e^{-2x}\,dx & \frac{1}{3}\sin(3x) \end{array}$$

$$= A + \frac{2}{3}\left[-\frac{1}{3}e^{-2x}\cos(3x) - \frac{2}{3}\int e^{-2x}\cos(3x)\,dx\right]$$

$$\begin{array}{c|c} e^{-2x} & \sin(3x)\,dx \\ \hline -2e^{-2x}\,dx & -\frac{1}{3}\cos(3x) \end{array}$$

$$= A - \frac{2}{9}e^{-2x}\cos(3x) - \frac{4}{9}\int e^{-2x}\cos(3x)\,dx$$

$$\Rightarrow \left(1 + \frac{4}{9}\right)\int e^{-2x}\cos(3x)\,dx = A - \frac{2}{9}e^{-2x}\cos(3x)$$

$$\int e^{-2x}\cos(3x)\,dx = \frac{9}{13}\left[A - \frac{2}{9}e^{-2x}\cos(3x)\right] + C$$

i) $\int \frac{23x^2 - 11x + 43}{(x-3)(3x^2+4)}\,dx = \int\left(\frac{A}{x-3} + \frac{Bx + C}{3x^2+4}\right)dx$

$$23x^2 - 11x + 43 = A(3x^2+4) + (x-3)(Bx+C)$$
$$= (3A+B)x^2 + (C-3B)x + 4A - 3C$$

$$\begin{cases} 3A + B = 23 \\ C - 3B = -11 \\ 4A - 3C = 43 \end{cases}$$

$A\Big|_{x=3} = \frac{207 - 33 + 43}{27 + 4} = 7$

$3(7) + B = 23 \Rightarrow B = 2$

$C - 6 = -11 \Rightarrow C = -5.$

$$= \int\left(\frac{7}{x-3} + \frac{2x-5}{3x^2+4}\right)dx$$

$$= \int\left[\frac{7}{x-3} + \frac{2}{6}\cdot\frac{6x}{3x^2+4} - \frac{5}{3}\cdot\frac{1}{\frac{4}{3} + x^2}\right]dx$$

$$= 7\ln|x-3| + \frac{1}{3}\ln(3x^2+4) - \frac{5}{4}\cdot\frac{\sqrt{3}}{2}\cdot\tan^{-1}\left(\frac{\sqrt{3}}{2}x\right) + C$$

j) $$\int \frac{3x-4}{\sqrt{24-6x-9x^2}}\,dx = \int \frac{3x+1-5}{\sqrt{25-(3x+1)^2}}\,dx$$

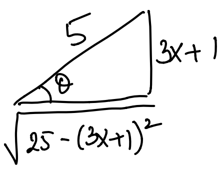
$$\left.\begin{aligned} 24-6x-9x^2 &= 24-\left[(3x)^2+2(3x)+1\right]+1 \\ &= 25-(3x+1)^2 \end{aligned}\right\} \quad \begin{aligned} &\text{let } 3x+1 = 5\sin\theta \\ &3dx = 5\cos\theta\,d\theta \Rightarrow dx = \tfrac{5}{3}\cos\theta\,d\theta \end{aligned}$$

$$\int \frac{5\sin\theta - 5}{\sqrt{25-25\sin^2\theta}}\cdot\frac{5}{3}\cos\theta\,d\theta = \frac{25}{3}\int \frac{\sin\theta - 1}{\sqrt{25\cos^2\theta}}\cos\theta\,d\theta$$

$$= \frac{5}{3}\int(\sin\theta - 1)\,d\theta = \frac{5}{3}\left[-\cos\theta - \theta\right] + C$$

from $3x+1 = 5\sin\theta$

$$\Rightarrow \sin\theta = \frac{3x+1}{5} \Rightarrow \theta = \sin^{-1}\left(\frac{3x+1}{5}\right)$$



$$\text{Ans} = -\frac{5}{3}\left[\frac{\sqrt{25-(3x+1)^2}}{5} + \sin^{-1}\left(\frac{3x+1}{5}\right)\right] + C$$