Quiz #2 **Math 181** **Name:** KEY

6/30/21

Show all your work clearly. No Work, No Credit.

1. Integrate the following:

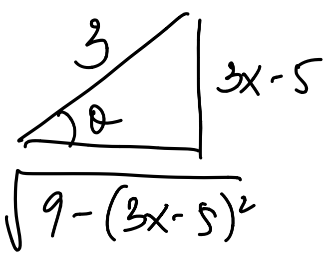
a) $\displaystyle\int \frac{3x+7}{\left(30x-16-9x^2\right)^{3/2}}dx = \int \frac{3x-5+12}{\left[9-(3x-5)^2\right]^{3/2}}dx$

$$\left\{\begin{aligned} 3x-5 &= 3\sin\theta \\ 3\,dx &= 3\cos\theta\, d\theta \\ dx &= \cos\theta\, d\theta \end{aligned}\right.$$

$$30x-16-9x^2 = -\left[(3x)^2 - 2(3x)\cdot 5 + 25\right] + 25 - 16 = 9 - (3x-5)^2$$

$$\Rightarrow \int \frac{3\sin\theta + 12}{\left[9-9\sin^2\theta\right]^{3/2}}\cos\theta\, d\theta = 3\int \frac{\sin\theta+4}{9^{3/2}\cdot\cos^3\theta}\cos\theta\, d\theta = \frac{1}{9}\int \frac{\sin\theta+4}{\cos^2\theta}d\theta$$

$$= \frac{1}{9}\int\left(\tan\theta\sec\theta + 4\sec^2\theta\right)d\theta = \frac{1}{9}\left[\sec\theta + 4\tan\theta\right] + C$$



$$\Longrightarrow \frac{1}{9}\left[\frac{3}{\sqrt{9-(3x-5)^2}} + 4\,\frac{3x-5}{\sqrt{9-(3x-5)^2}}\right] + C$$

b) $\displaystyle\int \frac{x^2-15x+41}{(x+2)(x-3)^2}dx = \int\left[\frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}\right]dx$

$$A\Big|_{x=-2} = \frac{4+30+41}{25} = 3\ ; \quad C\Big|_{x=3} = \frac{9-45+41}{5} = 1.$$

$$\text{let } x=0 \Rightarrow \frac{41}{2(9)} = \frac{A}{2} - \frac{B}{3} + \frac{C}{9} = \frac{3}{2} - \frac{B}{3} + \frac{1}{9}$$

$$41 = 27 - 6B + 2 \Rightarrow B = \frac{29-41}{6} = -2.$$

$$\Rightarrow \int\left(\frac{3}{x+2} - \frac{2}{x-3} + \frac{1}{(x-3)^2}\right)dx = 3\ln|x+2| - 2\ln|x-3| + \int\frac{1}{(x-3)^2}dx$$

$$\text{let } u = x-3 \Rightarrow du = dx$$

$$= \ln\left(\frac{(x+2)^3}{(x-3)^2}\right) + \int\frac{1}{u^2}du = \ln\left(\frac{(x+2)^3}{(x-3)^2}\right) - \frac{1}{x-3} + C$$

2. Sketch and then set up integral(s) for volumes of the region bounded by the curves:
$y=-x^2-2x+3$ *and* $y=x-7$ which is rotated about the following lines:

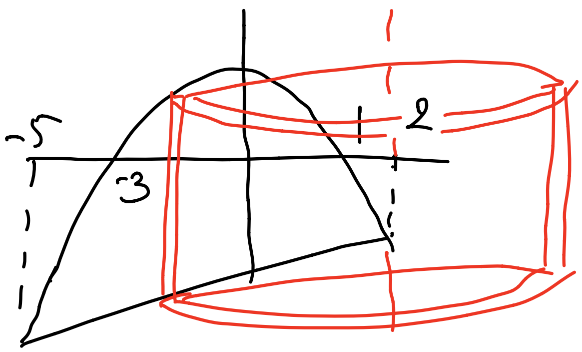
a) $x=2$

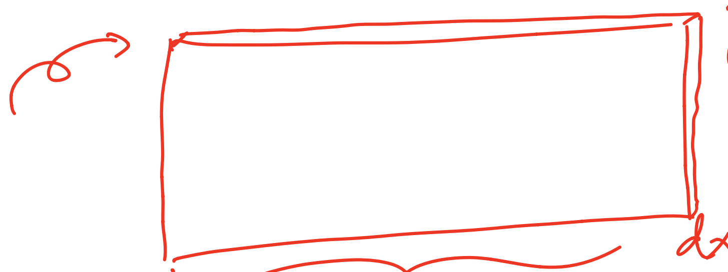
$$y=-(x^2+2x-3)=-(x+3)(x-1)$$

pts of intersection: $-x^2-2x+3=x-7$

$x^2+3x-10=0$

$(x+5)(x-2)=0 \Rightarrow x=-5,2.$





$$\left.\begin{array}{l}-x^2-2x+3\\-(x-7)\\=-x^2-3x+10\end{array}\right.$$

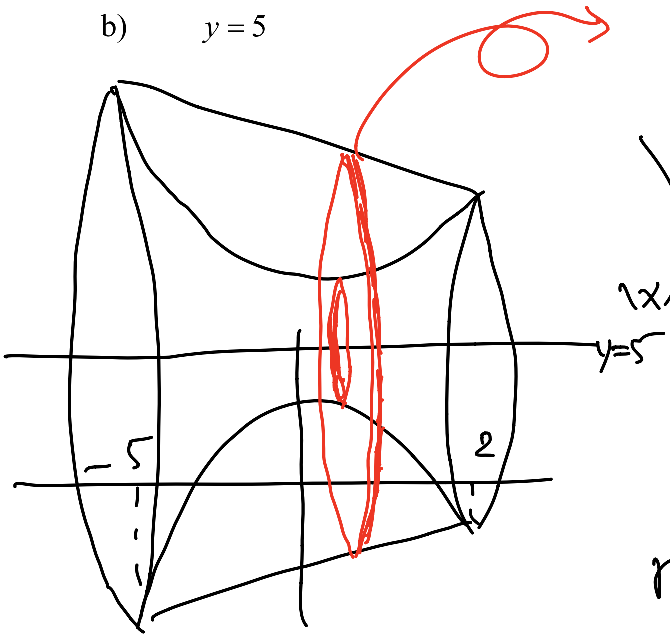
dx

$2\pi r=2\pi(2-x)$

$$V=2\pi\int_{-5}^{2}(2-x)(10-3x-x^2)\,dx$$

b) $y=5$

dx



$$V=\pi\left[R_o^2-r_i^2\right]dx.$$

where

$R_o = $ (from $y=5$ to $y=x-7$) $=5-(x-7)$

$=12-x$

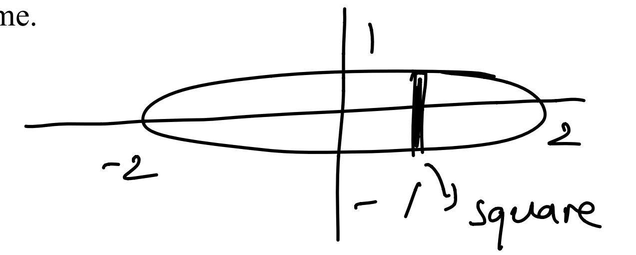
$r_i = $ (from $y=5$ to $y=-x^2-2x+3$) $=5-(-x^2-2x+3)$

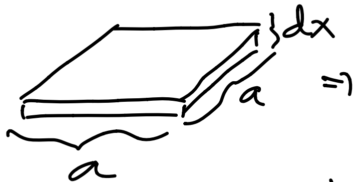
$=x^2+2x+2.$

$$\Rightarrow V=\pi\int_{-5}^{2}\left[(12-x)^2-(x^2+2x+2)^2\right]dx$$

3. The base of a solid is bounded by $R=\{(x,y)\,|\,x^2+4y^2\le 4\}$. All parallel – sections are squares perpendicular to the base and the x – axis. Find its volume.

$$x^2+4y^2=4 \Rightarrow \frac{x^2}{4}+y^2=1$$



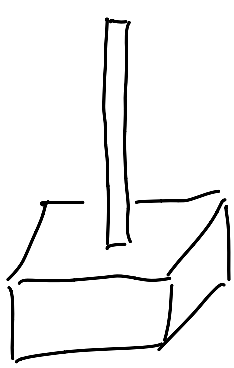


$\Rightarrow V=a^2dx$ where $a=\text{II}=2\sqrt{1-\frac{x^2}{4}}$

$$\Rightarrow V=\int_{-2}^{2}\left(2\sqrt{1-\frac{x^2}{4}}\right)^2dx = 2\int_0^2 4\left(1-\frac{x^2}{4}\right)dx.$$

$$=8\left[x-\frac{1}{12}x^3\right]_0^2 = 8\left[2-\frac{2}{3}\right]=\boxed{\frac{32}{3}}$$

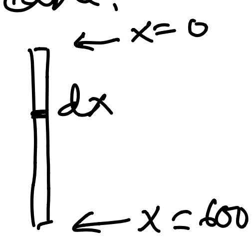
4. A 600 – ft – cable that weights 300lbs is attached to a 2000 – lbs – elevator. Calculate the total work required to pull both the cable and the elevator up 600 ft. (10pts)



For the elevator: $W_e = F\cdot d = (2000\text{ lbs})(600\text{ ft})$

$= 1200000$ ft-lbs.

For the cable:



600ft → weights 300 lbs.

$\Rightarrow$ 1ft → weight: $\frac{1}{2}$ lb.

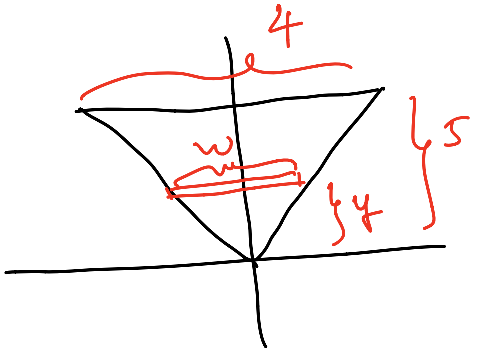
$\Rightarrow F=\frac{1}{2}dx$; $d=x$.

$$W_c=\int_0^{600}\frac{1}{2}x\,dx = \frac{1}{4}x^2\Big|_0^{600} = \frac{360000}{4}=90{,}000 \text{ ft-lbs}$$

Total work $= W = W_e + W_c = 1{,}200{,}000+90{,}000$

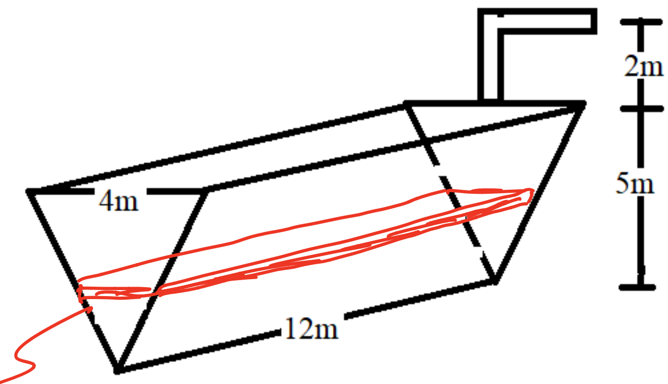
$= 1{,}290{,}000$ ft-lbs.

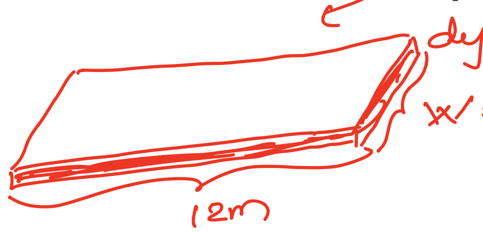
5. The following tank is full of water. Determine the work required to pump the water out of the outlet.



$\frac{w}{4} = \frac{y}{5}$

$w = \frac{4}{5}y$





$\Rightarrow V = 12 \cdot \frac{4}{5}y \cdot dy$

$F = 9800 \cdot \frac{48}{5} \cdot y \cdot dy$

$d = 7 - y$

$W = 94{,}080 \int_0^4 y \cdot (7-y)\,dy$

$= 94{,}080 \cdot \left[\frac{7}{2}y^2 - \frac{1}{3}y^3\right]_0^4$

$= 94{,}080 \left[\frac{7}{2}(16) - \frac{1}{3}(64)\right]$

$= 3{,}261{,}440 \text{ J}.$